Symmetry breaking in your kitchen sink

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Investigations of curiosities of everyday life physics can sometimes start in as innocuous a place as your kitchen sink – in there you have probably noticed the so-called hydraulic jump: a sudden jump in fluid height is formed in the outward spreading water layer at the bottom of the sink when you turn on the tap. This jump usually appears on a more or less circular locus around the impact point of the water jet.

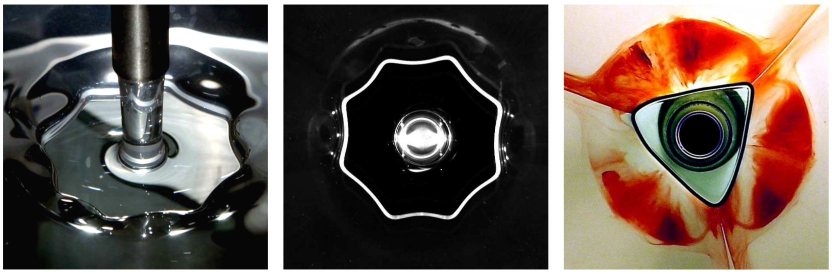

Figure 1: Hydraulic jump with eight corners, shown from an oblique angle (left) and from below (center and right). The "roller" flow structure is visualized with red dye in the triangle (right).

What you probably haven't yet seen in your sink is that the jump can turn into a "polygon": In 1997, researchers in Copenhagen discovered that the jump may undergo structural changes where the circular jump symmetry is broken, corners develop, and the jump shape becomes polygonal. To transform into such a polygonal state, the system first has to change from a "Type I" jump to a "Type II" jump. In the Type I state all surface velocities point outward. A transition to the "Type II" state occurs when the liquid layer outside the jump is sufficiently deep; in this state, there is an annular region outside the jump, where the surface flow points inward: a so-called "roller", i.e. an eddy structure similar to a breaking wave, has appeared. The original explanation for the polygonal shapes was based on a competition between the outward viscous forces and the inward gravity on this roller, but the origin of the instability breaking the circular symmetry remained unclear. A significant step towards finding an explanation for the instability was made by John Bush and coworkers in 2006, who pointed out that surface tension plays a major role and that the mechanism causing polygons is similar to the so-called Rayleigh-Plateau instability. This instability is responsible for the breaking-up of long water jets into droplets – and in the case of the jump, it leads to the breaking-up of the roller. Martens, Watanabe and Bohr present a comprehensive model, which combines these two aspects. Their starting point is again the torus-shaped roller on which forces act (including the Laplace pressure due to surface tension) and inside which there can be slow fluid transport. This model leads to shapes that look surprisingly close to those observed in experiments, and to an instability criterion that relates a shape parameter to a "Bond number" (the ratio of the typical length scales in the problem to the capillary length). Instability will set in on a length scale which is roughly proportional to the roller width – analogously to the Rayleigh-Plateau instability and in agreement with experiments.

It is surprising that a "simple" flow like this is actually so difficult to analyze. It is known that the Navier-Stokes equations are hard to solve, but since the hydraulic jump flows are laminar, one would expect that at least a numerical solution should be available. However, when a strongly deformed free surface is involved, the numerical solution becomes very hard, and to date no 3D-simulations have been made of the polygonal hydraulic jump. From a theoretical point of view, these flows contain several of the classic fluid problems. The jump itself is mainly driven by flow separation (where part of the flow goes backward), and in the Type II state the separation occurs at the fluid surface. It is well known that Prandtl's boundary layer equations – which are the natural starting point for the analysis – break down at such points, and this has made progress very difficult. For the Type I state it was shown by Watanabe and coworkers in 2003 that one actually can continue Prandtl's equations through the jump by taking averages (in analogy with the so-called Karman-Pohlhausen approach), but this has not been done for the Type II state, from which the polygons develop.

The full flow is very complicated, as seen in the triangle in the right-most figure, where red dye has been used to track the flow. Looking carefully one realizes that the roller breaks up at the corners, creating jet-like flows emanating from each of them. Here the basic assumption made by Martens et al. – that one can treat the roller as a deformed but continuous torus – obviously breaks down and new theoretical approaches have to be invented. A more detailed study of these processes will very likely lead to new unexpected phenomena and emphasize – again – that even laminar flows can be quite a challenge for theorists!

References:

- E. A. Martens, S. Watanabe, T. Bohr. "Model of the polygonal hydraulic jump". Phys. Rev. E Vol. 85, 036316 (2012); arXiv:1111.4599
- Supplementary informations
- Figure appearing in Physical Review E Kaleidoscope