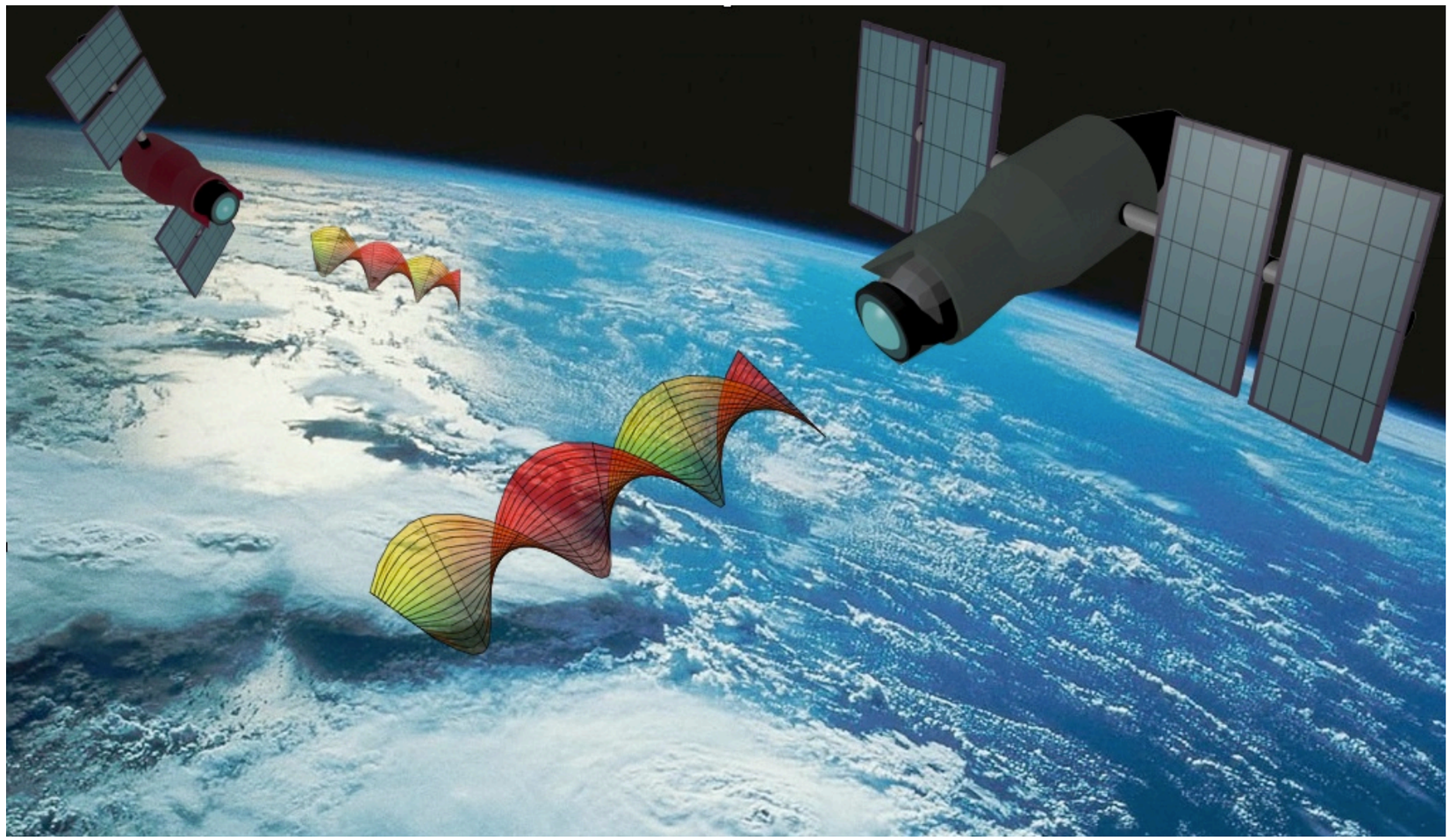
Hybrid single photon multidimensional systems as a resource for fundamental quantum mechanics and quantum communication



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SAPIENZA
UNIVERSITÀ DI ROMA

phorbitech

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Qubit and Qudit

Quantum bit (qubit) - fundamental unit of (quantum) information

$$|\psi\rangle = \alpha|0\rangle + \beta|1\rangle$$

Orthogonal quantum states

Going to higher dimensions - Qudits

Dim (H) =2

$$|\psi\rangle = \alpha|0\rangle + \beta|1\rangle$$

Dim (H) =D

$$|\phi\rangle = \sum_{i}^{D} c_i|i\rangle$$

D-dimensional advantages

Quantum Communication

- ✓Higher information density coding
- ✓Larger resilience to errors
- ✓Higher security in cryptography

Quantum Computation

- ✓Speed up computing tasks
- ✓More complex quantum computational architecture
- ✓New quantum algorithms

Quantum Simulations

- ✓Richer simulations of quantum mechanical systems

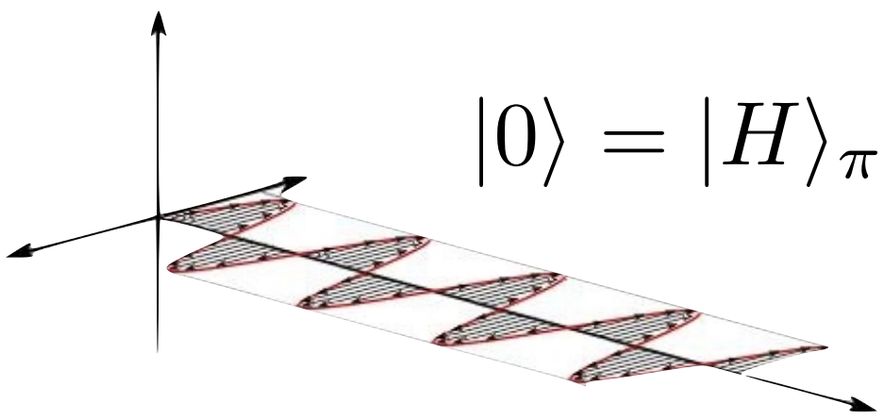
Fundamental Tests

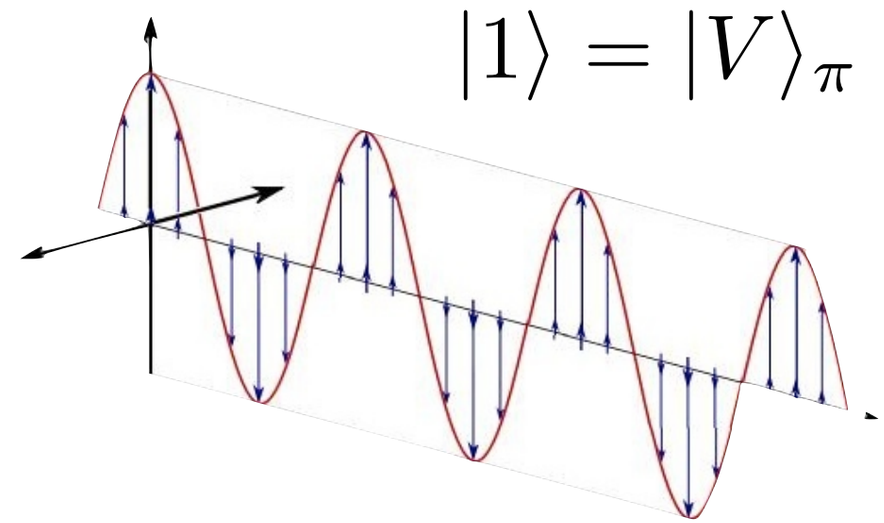
- ✓New tools for quantum foundation investigation
- ✓Stronger violations of Bell's like inequalities

Implementation

- **Qubit**

Usually implemented exploiting polarization of photons



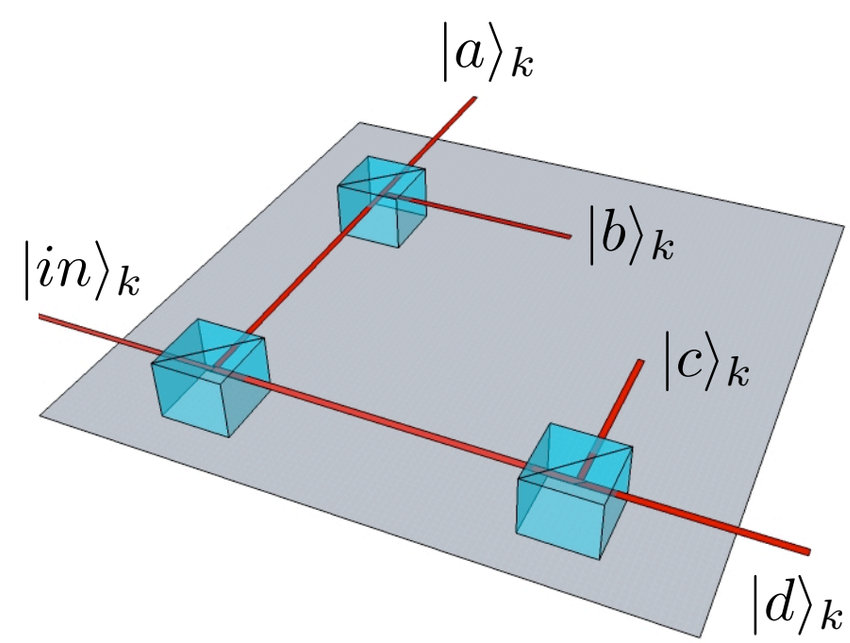


- **Qudit**

D>2 → Polarization is not enough

Multiphoton states or other degrees of freedom as:

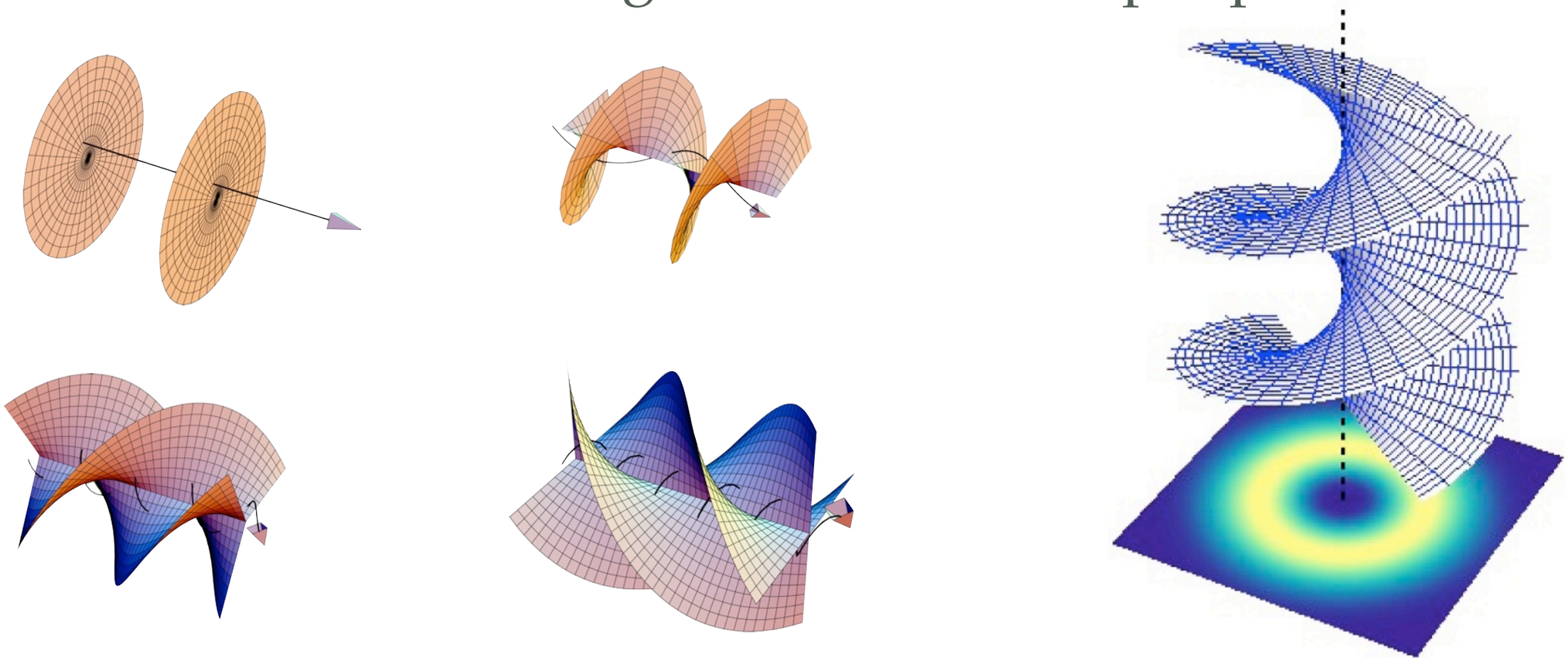
Linear momentum
Orbital angular momentum



Orbital angular momentum

Any wave with an azimuthal phase dependence $u(r, \phi, z) \propto e^{il\phi}$

carries $l\hbar$ of orbital angular momentum per photon



For instance, Laguerre-Gauss beams $|LG_{l.p}, \phi\rangle \propto e^{il\phi}$

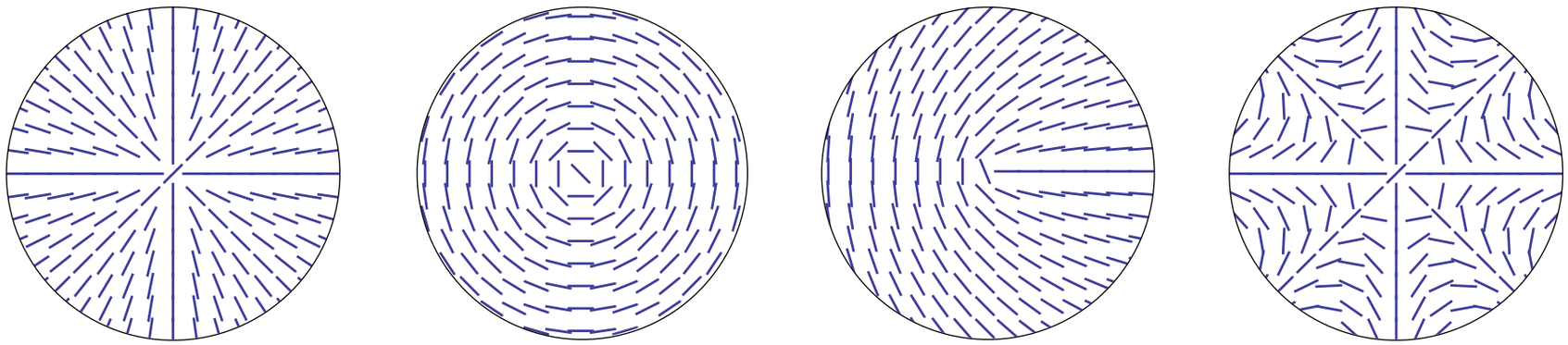
$l \in \mathbb{Z}$ ⇒ An infinite-dimensional Hilbert space to exploit

L. Allen et al Phys.Rev. A, 45(11):8185–8189, (1992).

Q-Plate

Inhomogeneous and birefringent

liquid crystals
director's orientation



$q = 1, \alpha = 0$ $\quad$ $q = 1, \alpha = \pi/2$ $\quad$ $q = 0.5, \alpha = 0$ $\quad$ $q = -3, \alpha = 0$

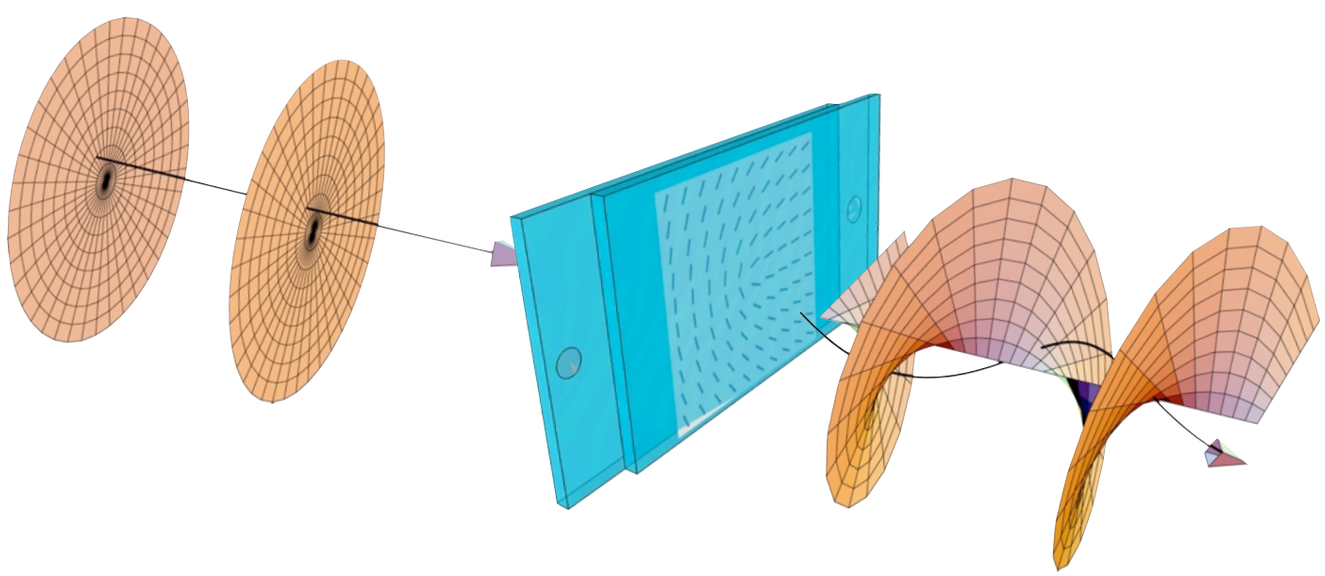
$$n(r, \phi) = q\phi + \alpha$$

$$QP|L\rangle_\pi = cos\left(\frac{\delta}{2}\right)|L\rangle_\pi + ie^{i2q\phi+2i\alpha}sin\left(\frac{\delta}{2}\right)|R\rangle_\pi$$

Tuned by an applied voltage

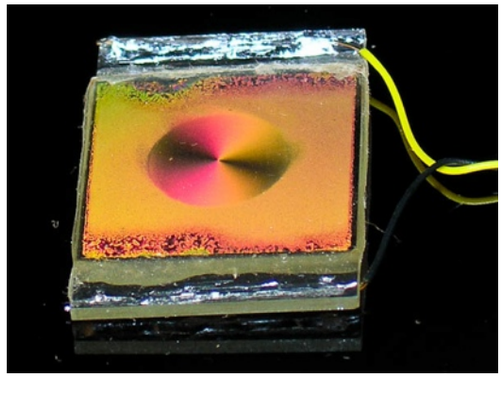
$$\alpha|L\rangle_\pi|0\rangle_{oam} + \beta|R\rangle_\pi|0\rangle_{oam}$$

$$\alpha|R\rangle_\pi|2q\rangle_{oam} + \beta|L\rangle_\pi|-2q\rangle_{oam}$$



L. Marrucci et al Phys.Rev. Lett, 96:163905, (2006).

Q-Plate (2)

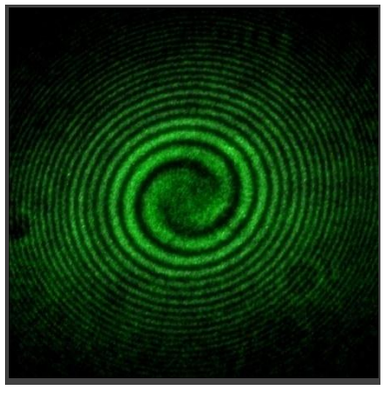


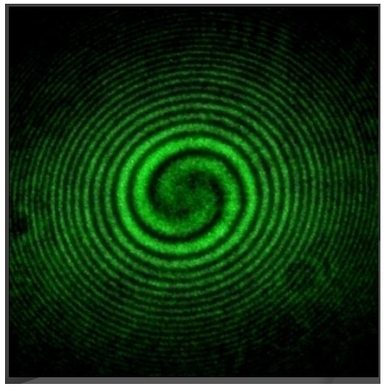
q=1

OAM eigenstates generation

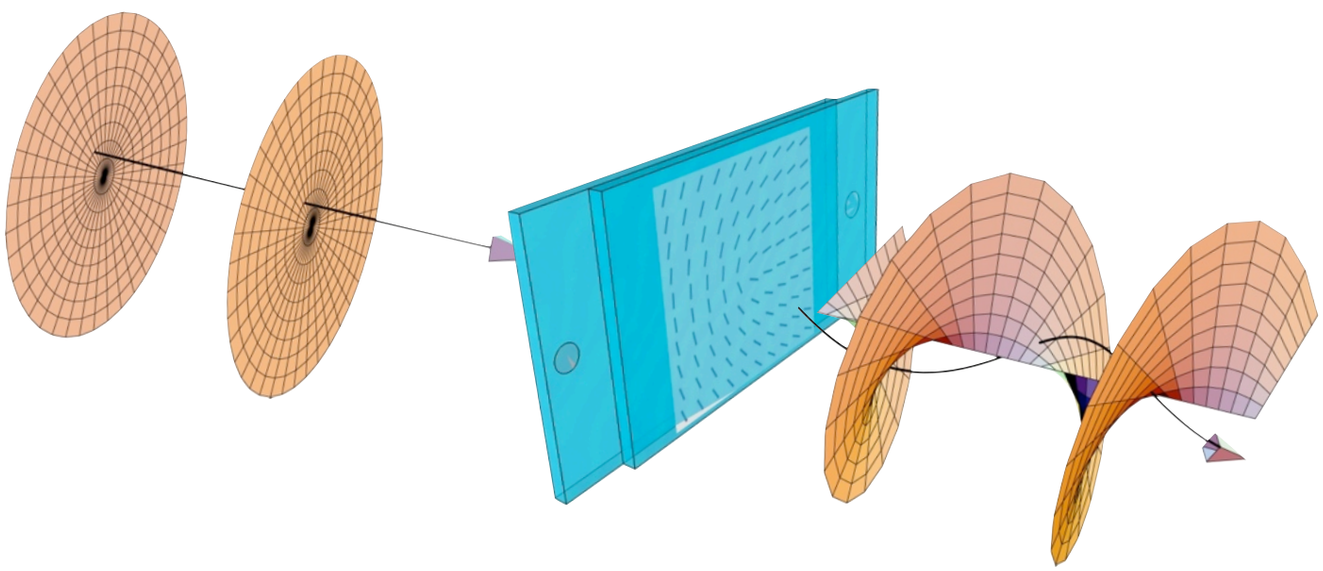
$$QP|R\rangle|0\rangle \rightarrow |L\rangle|-2\rangle$$

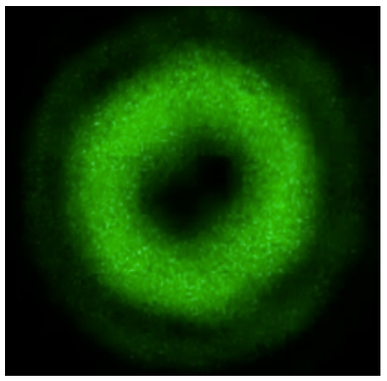
$$QP|L\rangle|0\rangle \rightarrow |R\rangle|+2\rangle$$





High fidelity, entanglement, good transmittance





Hybrid ququart space

D=4

Hybrid-ququart space Polarization and OAM

$$\{|H,+2\rangle, |H,-2\rangle, |V+2\rangle, |V-2\rangle\}$$

PRL **108,** 090501 (2012) PHYSICAL REVIEW LETTERS week ending 2 MARCH 2012

Experimental Observation of Impossible-to-Beat Quantum Advantage on a Hybrid Photonic System

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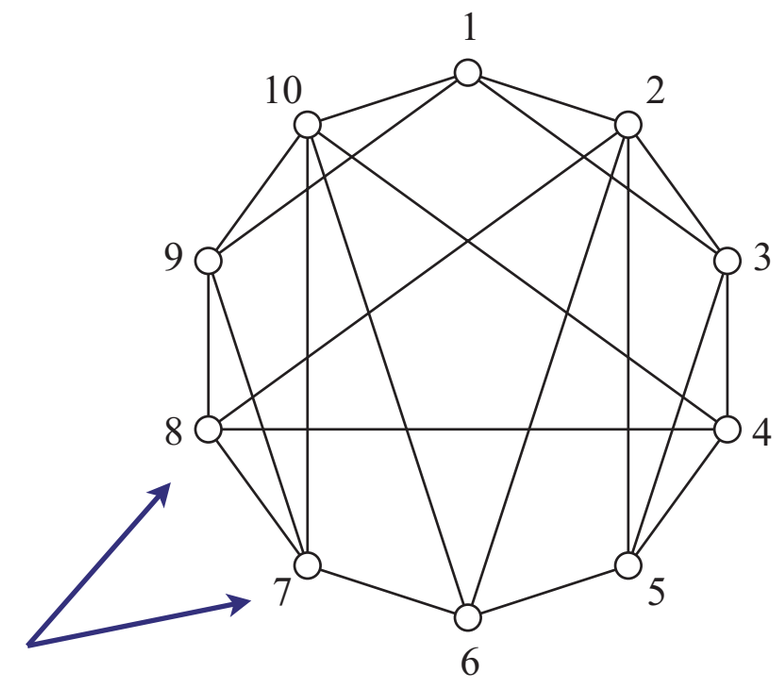
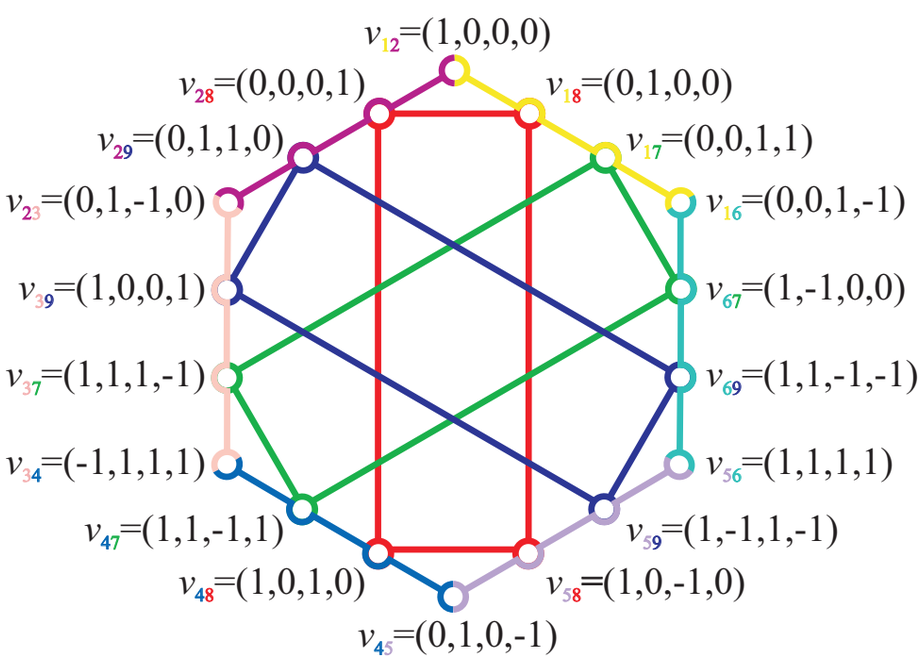
(Received 16 October 2011; published 27 February 2012)



orthogonal states



RAPID COMMUNICATIONS

PHYSICAL REVIEW A **84**, 030302(R) (2011)

Hybrid ququart-encoded quantum cryptography protected by Kochen-Specker contextuality

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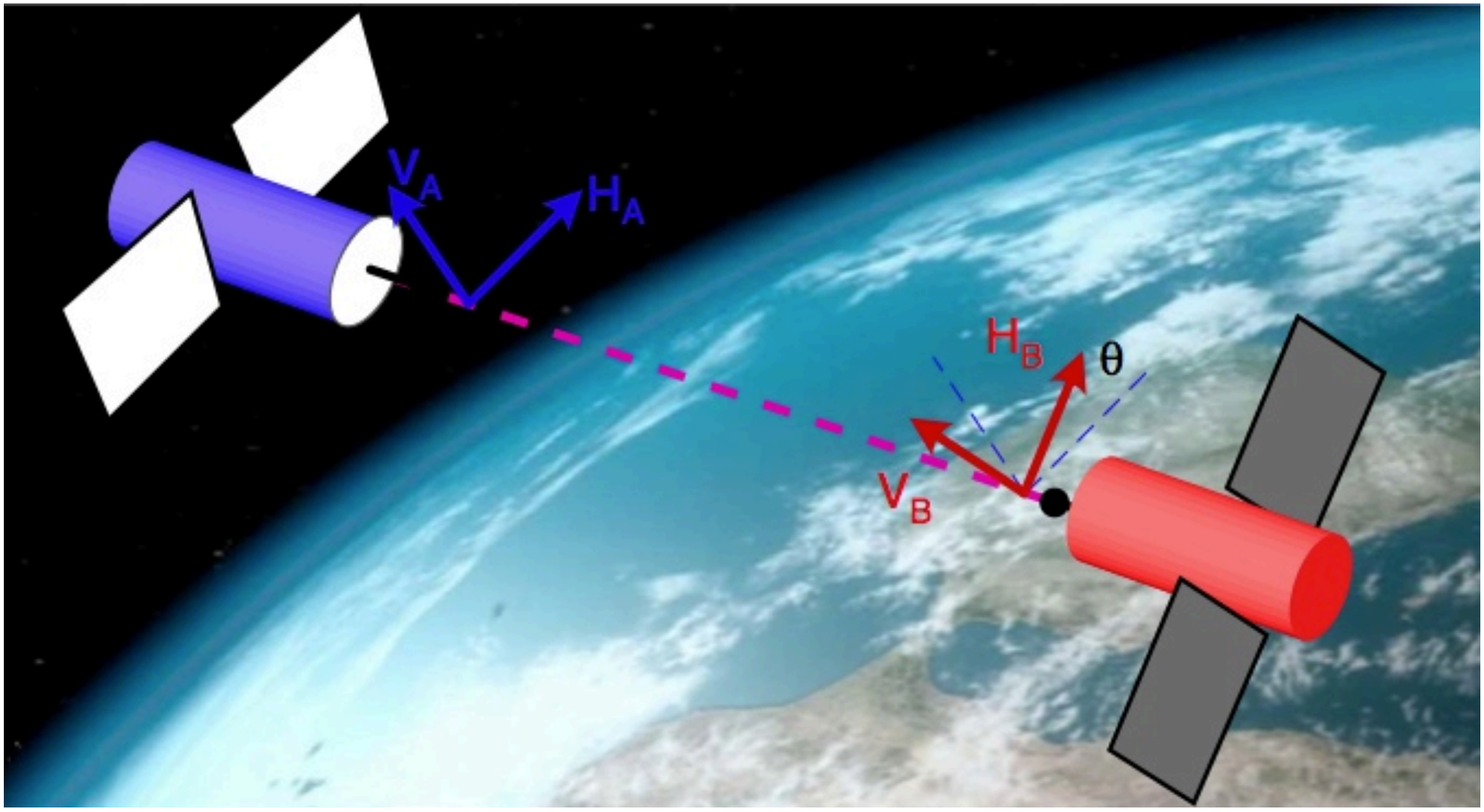
(Received 22 July 2011; published 12 September 2011)

Shared reference frame in Q.C.

Once Alice and Bob share an optical link they need to find θ in order to succesfully communicate



D'Ambrosio, V. *et al.* *Nat. Commun*. 3:961 doi: 10.1038/ncomms1951 (2012)

Rotational invariant single photon states

Hybrid ququart subspace

$$R[\theta] = \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix}$$

Polarization

$$R[\theta]|L\rangle = e^{-i\theta}|L\rangle$$

$$R[\theta]|R\rangle = e^{i\theta}|R\rangle$$

Orbital angular momentum

$$|LG_{l.p}, \phi\rangle \propto e^{il\phi}$$

$$|LG_{l.p}, \phi - \theta\rangle = e^{il\theta}|LG_{l.p}, \phi\rangle$$

with l=1 $\quad |+1\rangle = |l\rangle \quad |-1\rangle = |r\rangle$

$$R[\theta]|l\rangle = e^{-i\theta}|l\rangle$$

$$R[\theta]|r\rangle = e^{i\theta}|r\rangle$$

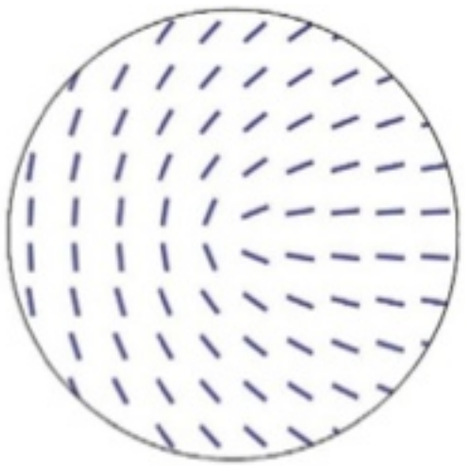
$$|0\rangle_L = |L\rangle_\pi |r\rangle_{o1}$$

$$|1\rangle_L = |R\rangle_\pi |l\rangle_{o1}$$

Implementation

$$R[\theta]|0\rangle_L = |0\rangle_L \qquad R[\theta]|1\rangle_L = |1\rangle_L$$

q=0.5



A Q-Plate with q=0.5 acts as a universal encoder/decoder

$$QP|R\rangle = |L\rangle|r\rangle = |0\rangle_L$$
$$QP|L\rangle = |R\rangle|l\rangle = |1\rangle_L$$

Encoding: $QP(\alpha|R\rangle + \beta|L\rangle) = \alpha|0\rangle_L + \beta|1\rangle_L$

Decoding: $QP(\alpha|0\rangle_L + \beta|1\rangle_L) = \alpha|R\rangle + \beta|L\rangle$

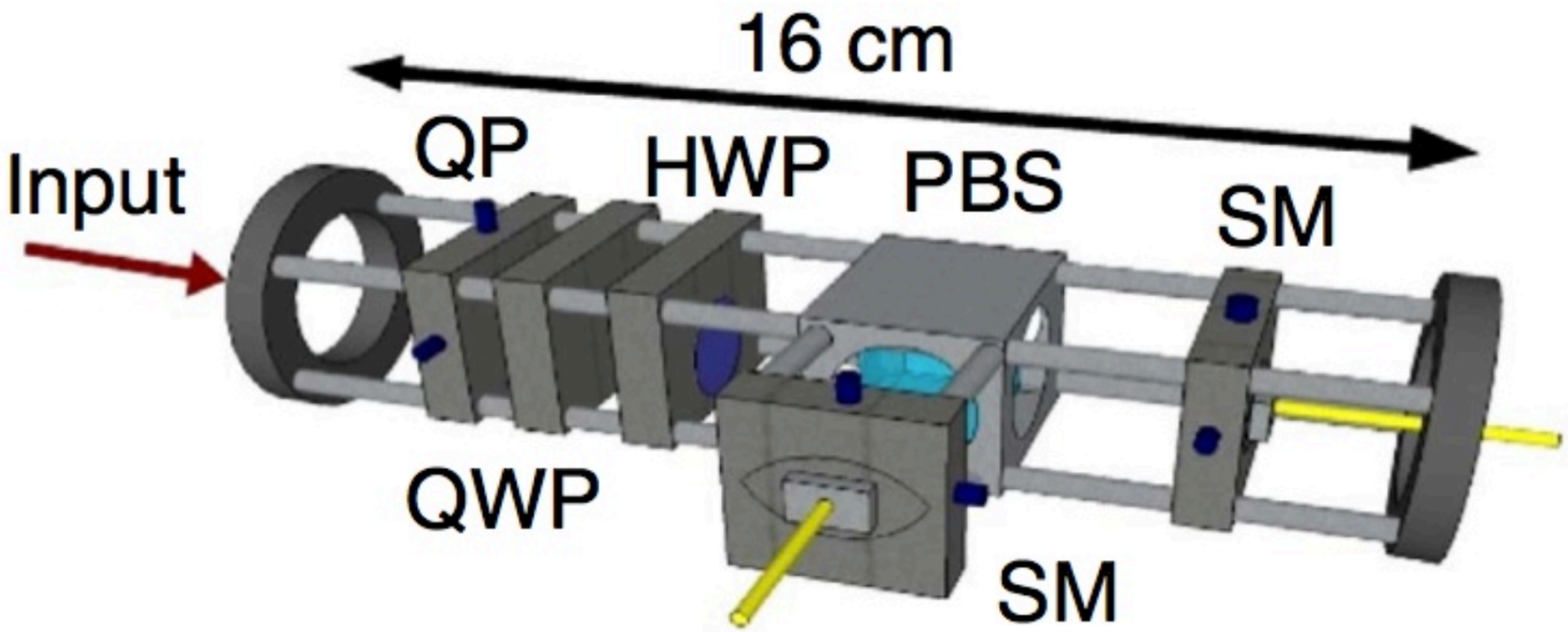
Polarization space $\xleftrightarrow{\quad QP \quad}$ Logical space

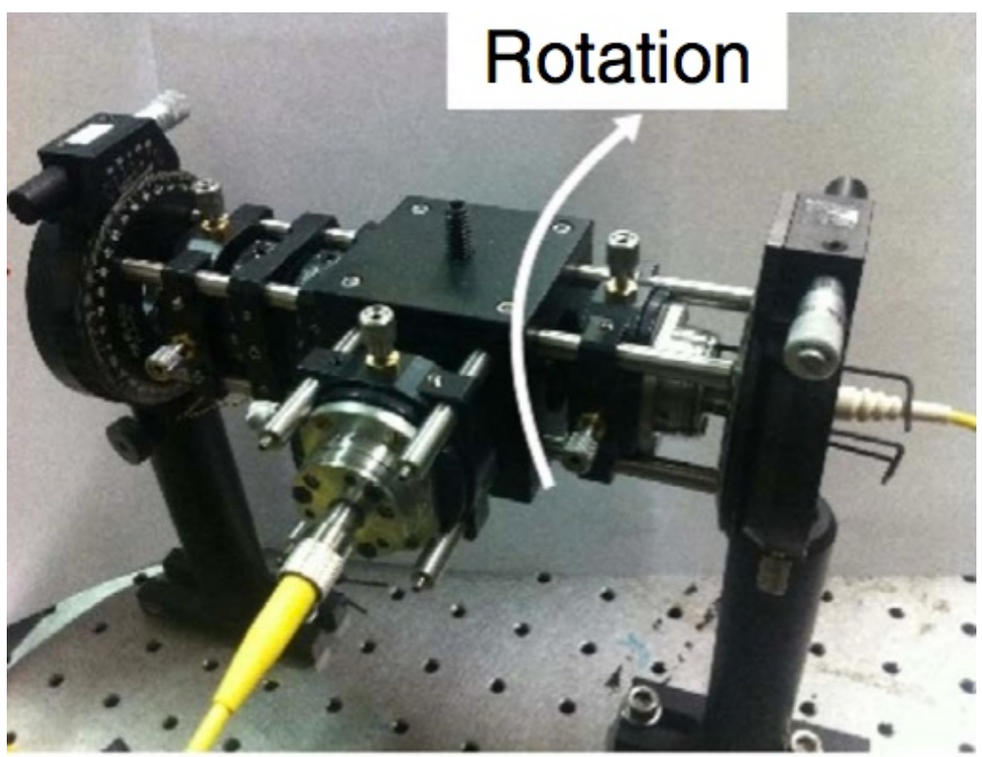
The experiment

We need a detection stage which is able to rotate

The experiment

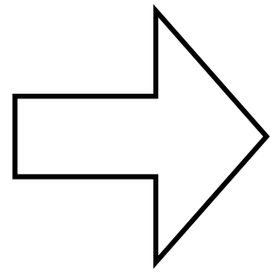
We need a detection stage which is able to rotate



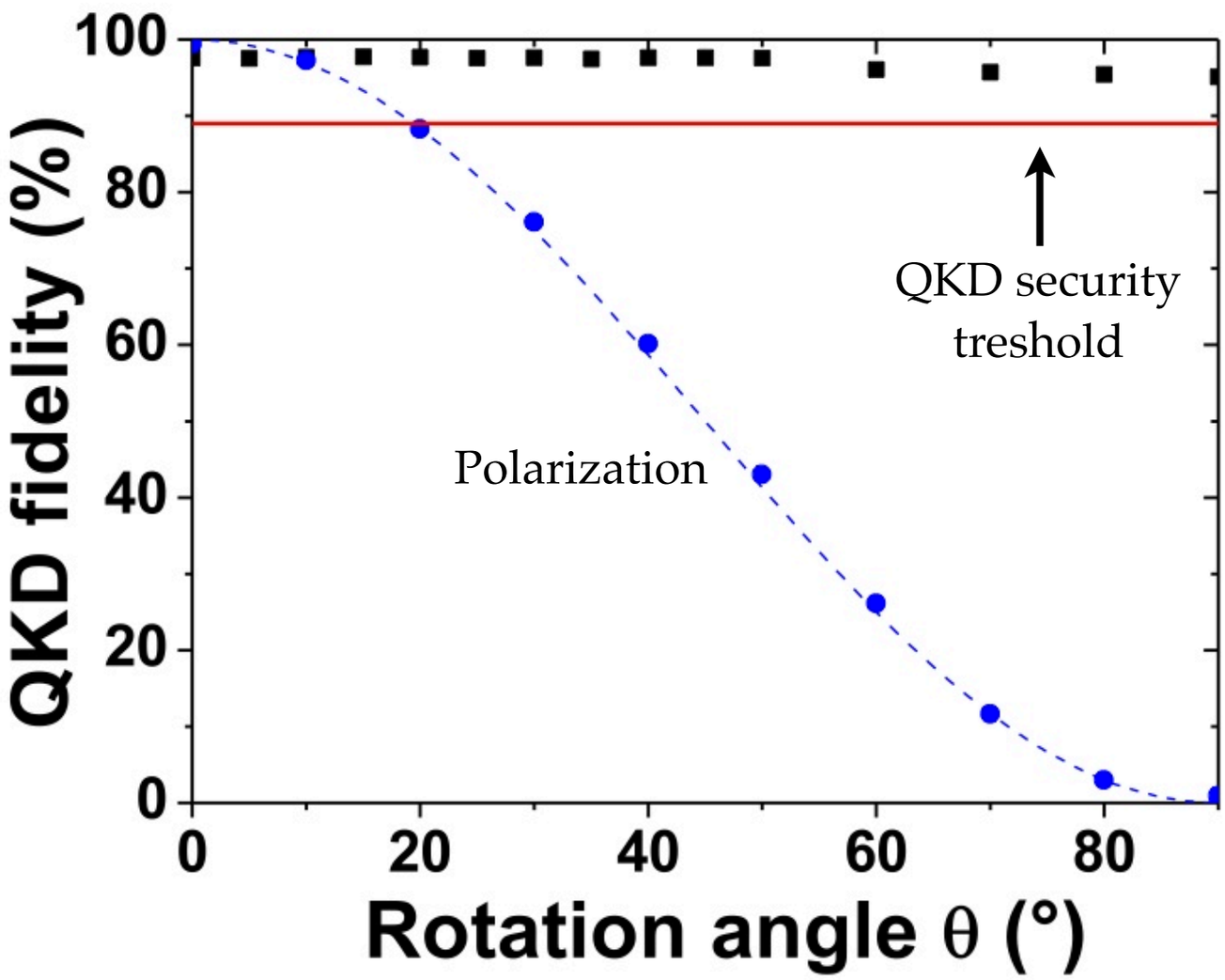


Alignement-free quantum key distribution

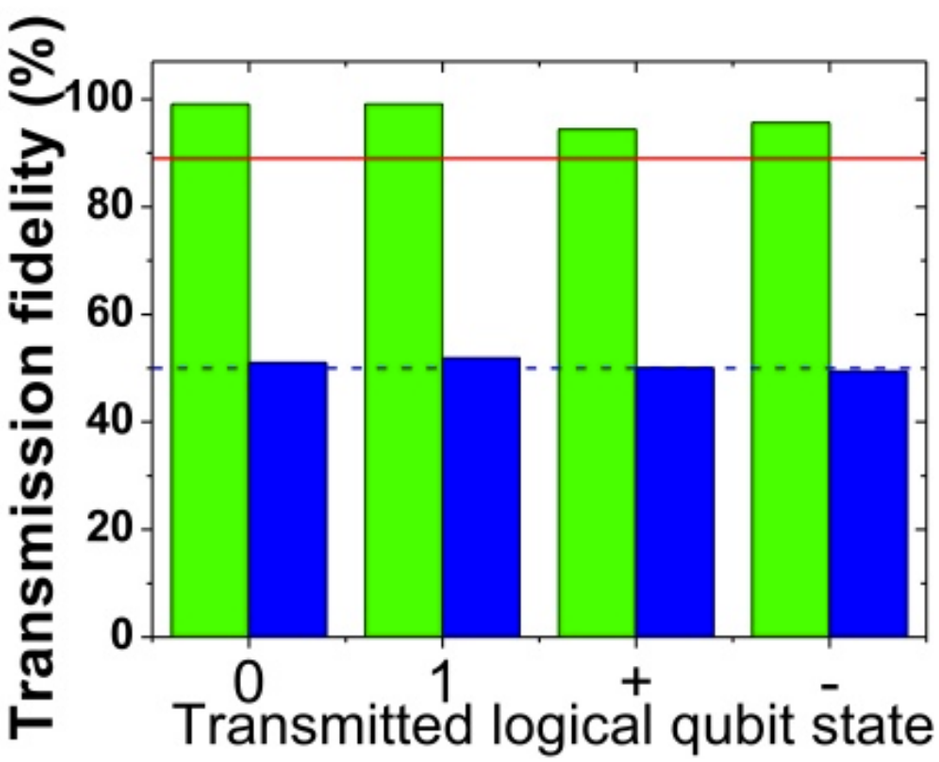
Bob encodes information



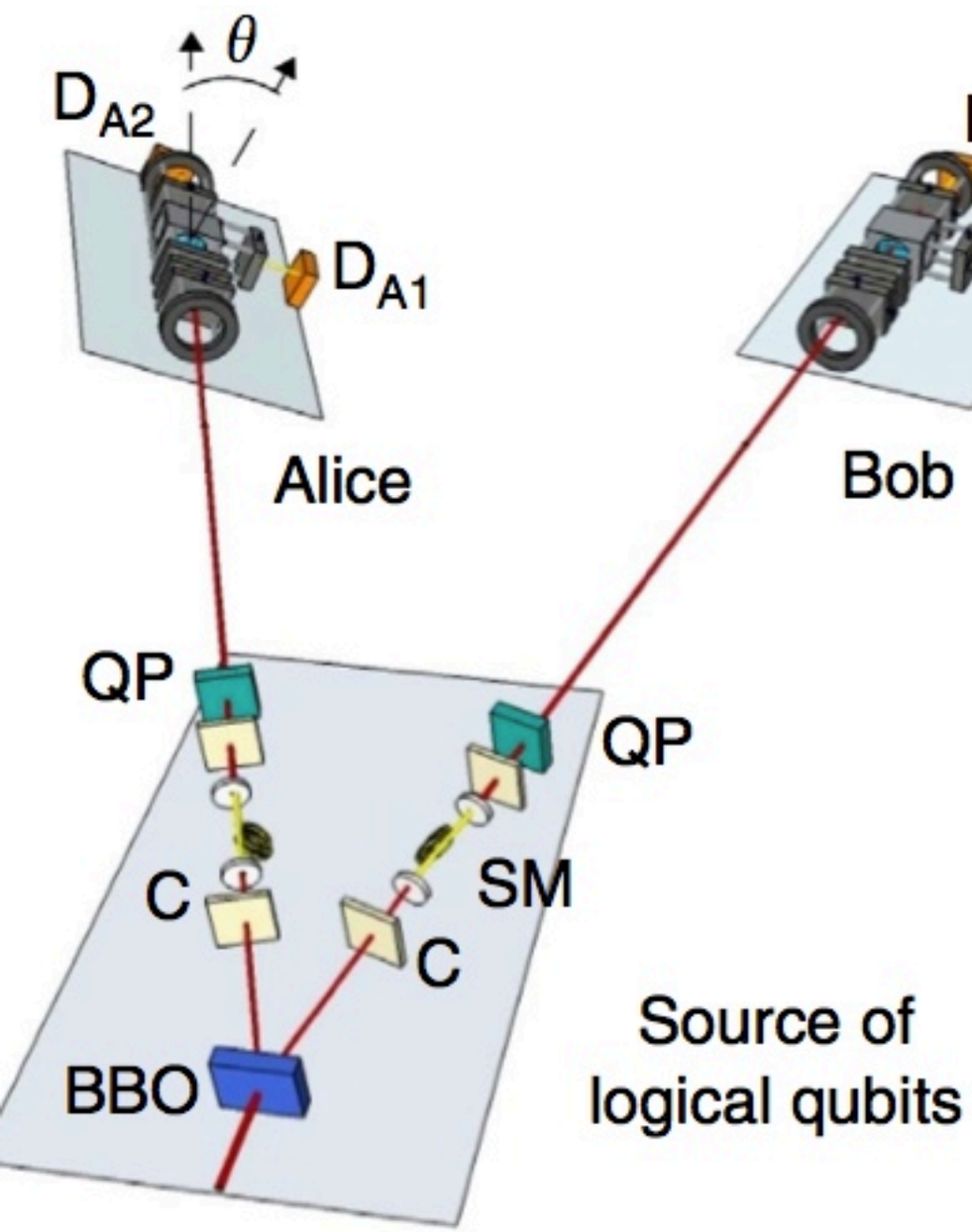
Alice decodes information for different rotation angles



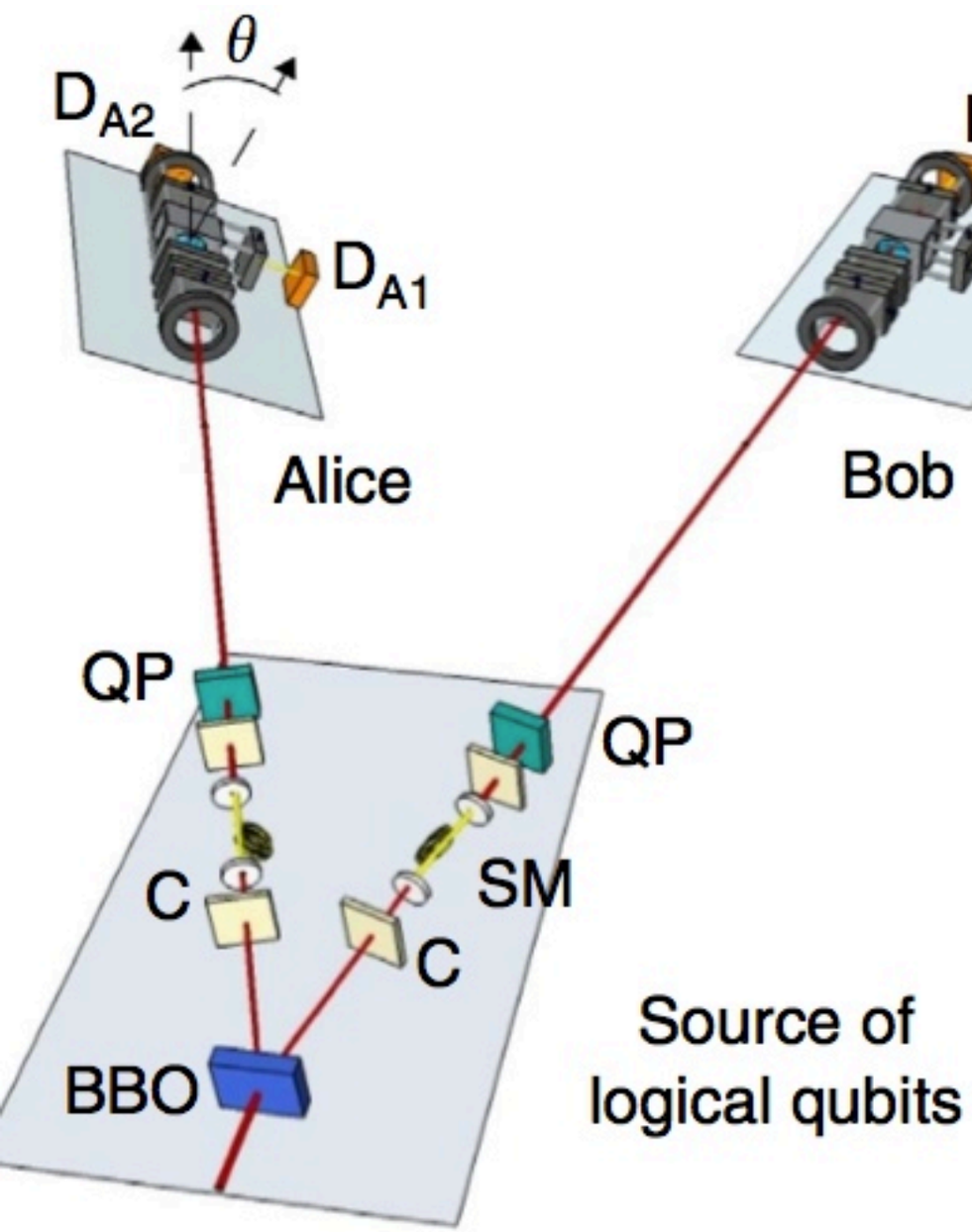
Average fidelity over a whole rotation session



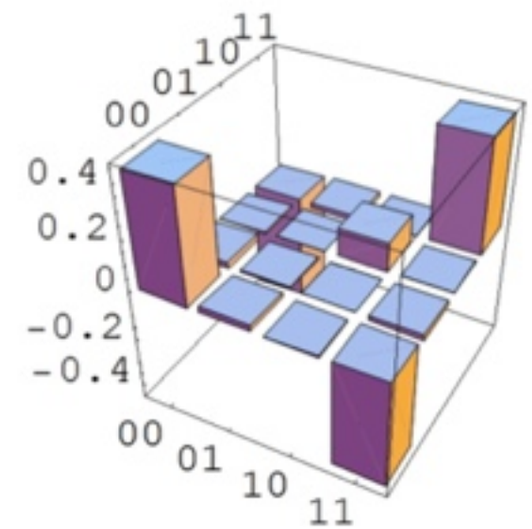
Non-locality test



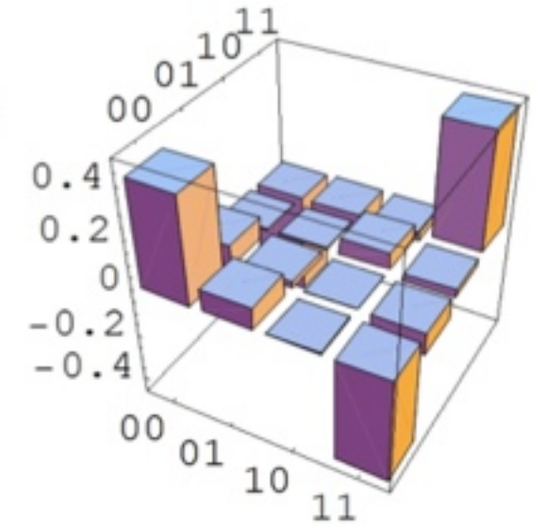
Non-locality test



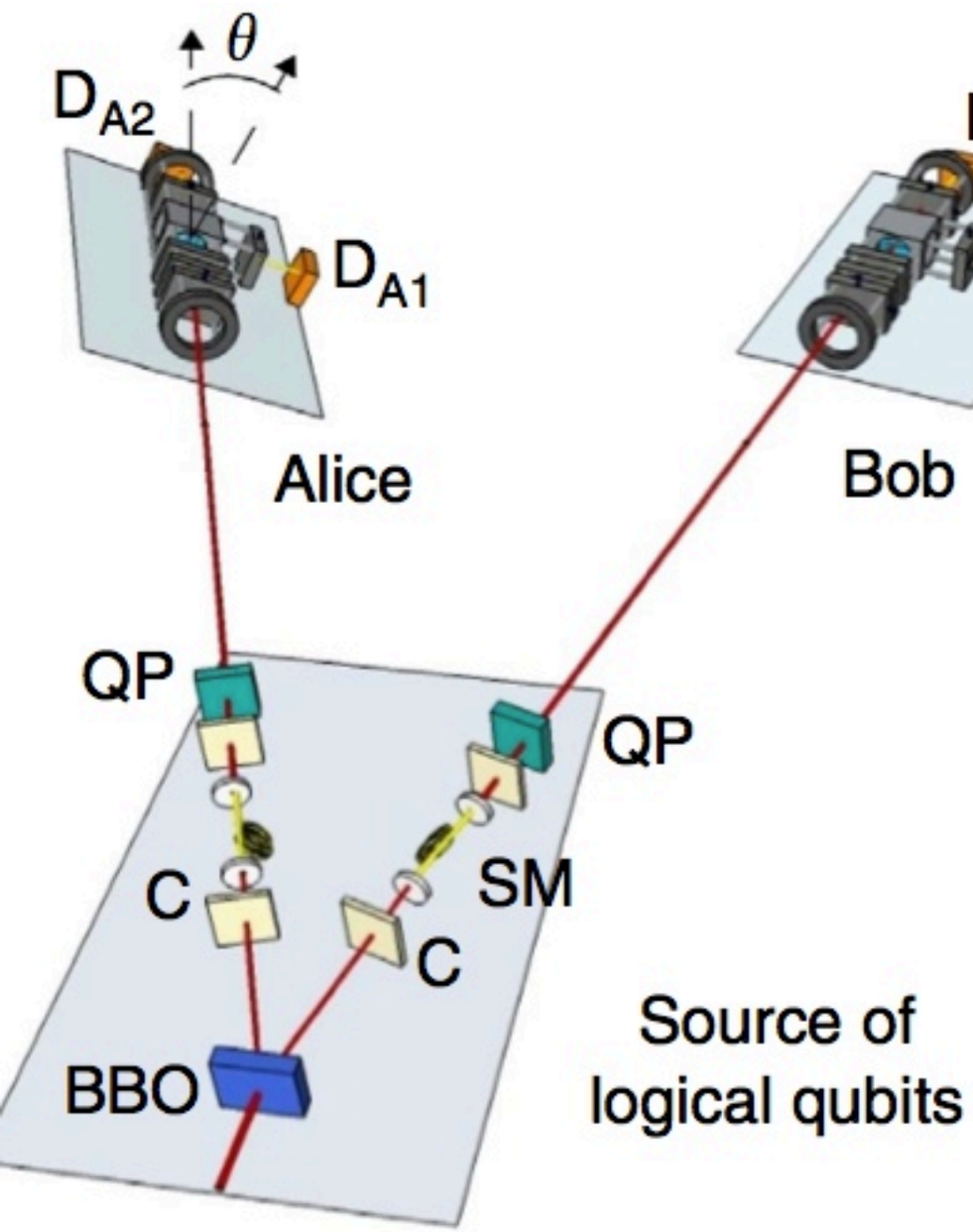
$\theta = 0$

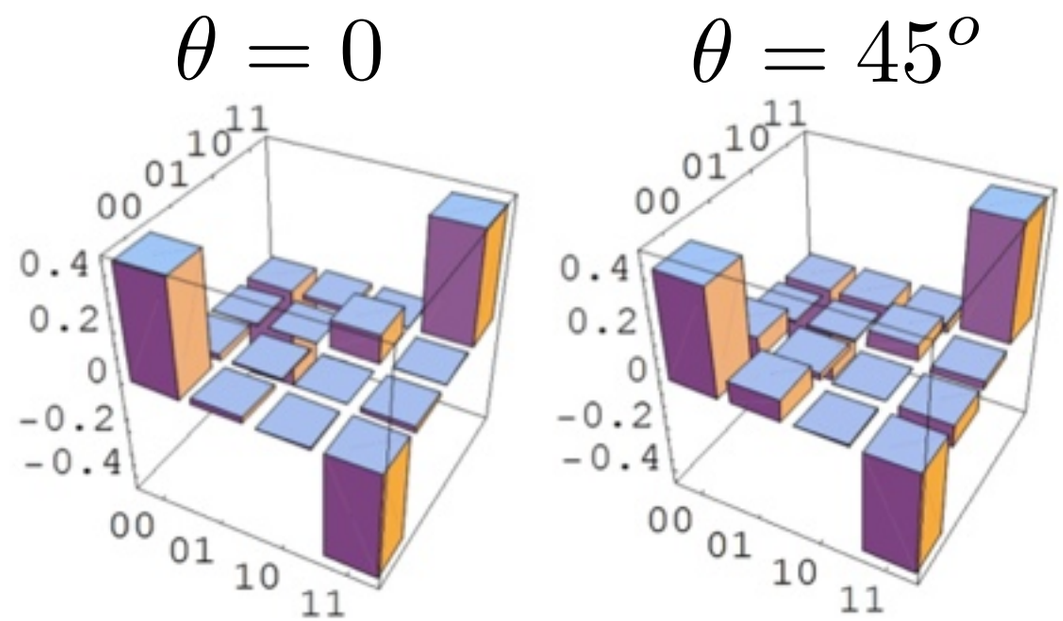


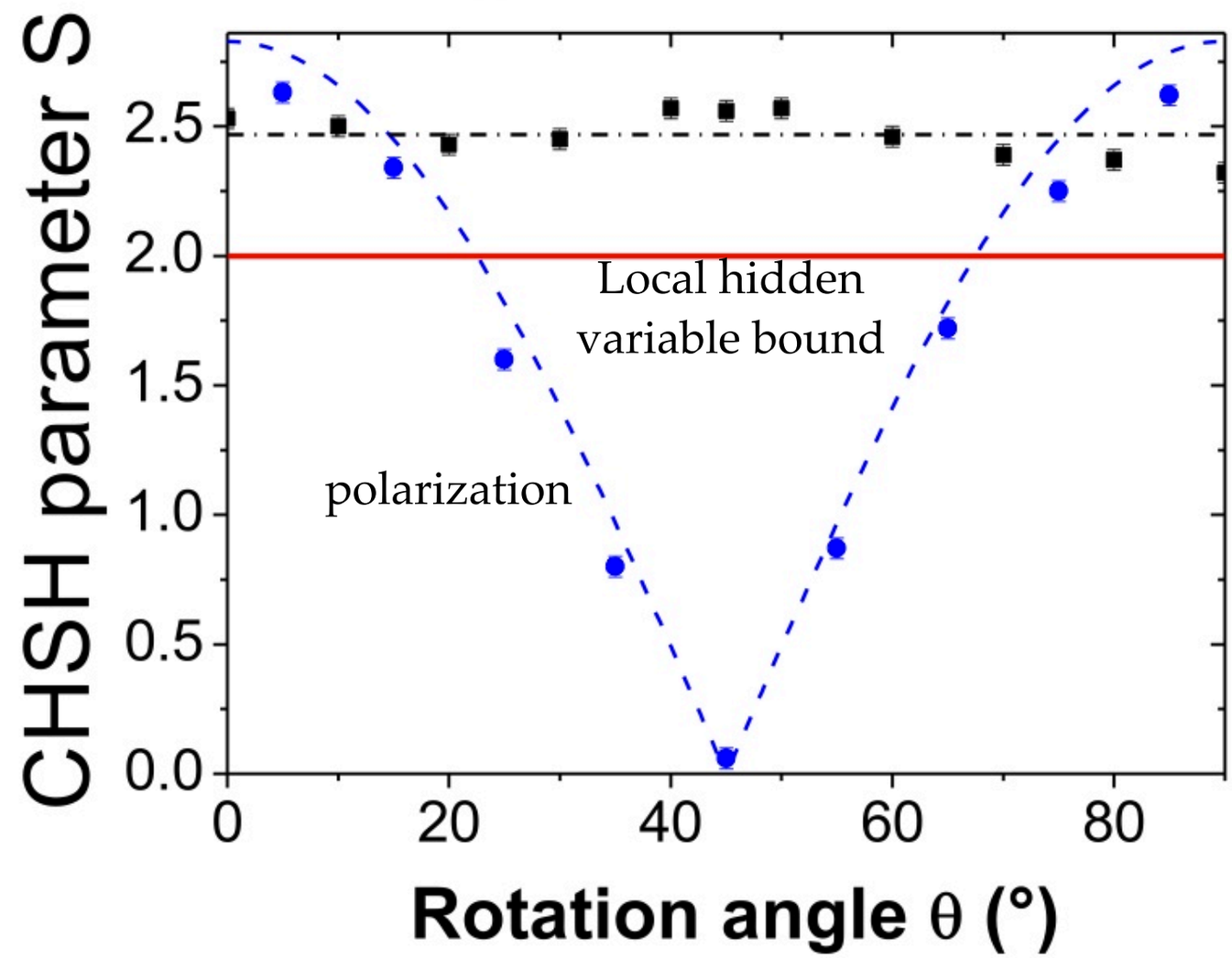
$\theta = 45^o$



Non-locality test







Robustness of rotational-invariant qubits

Bob's qubit $$|\psi\rangle = \alpha|R\rangle|l\rangle + \beta|L\rangle|r\rangle$$

What arrives to Alice after spatial mode perturbation $$\sum_m C_{+1,m}\alpha|R\rangle|m\rangle + C_{-1,m}\beta|L\rangle|m\rangle$$

After a Q-Plate $$\sum_m C_{+1,m}\alpha|L\rangle|m-1\rangle + C_{-1,m}\beta|R\rangle|m+1\rangle$$

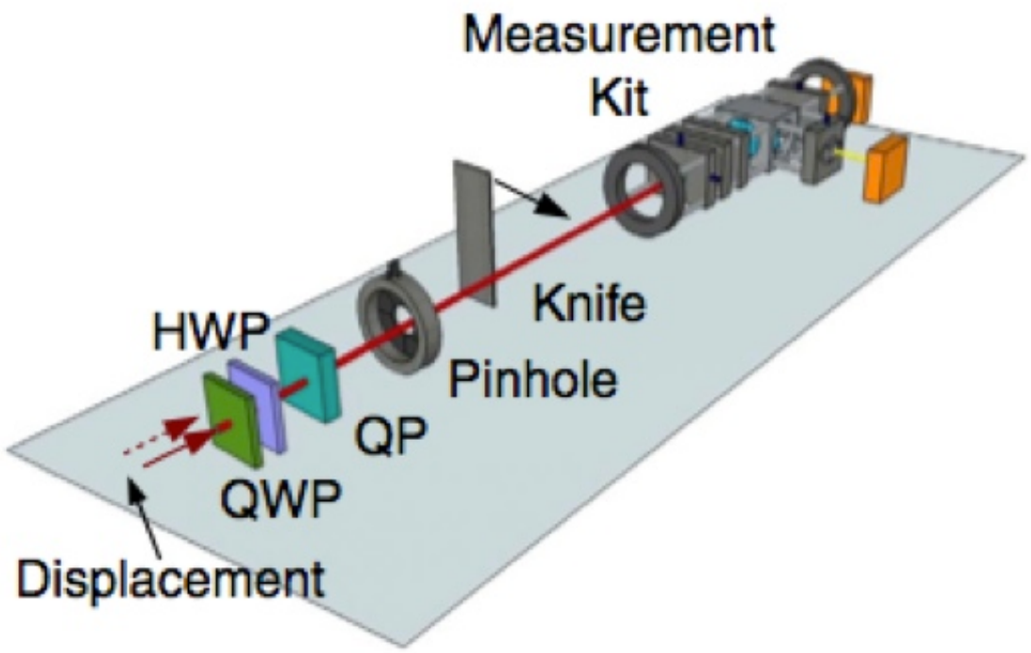
After a projection on mode $|0\rangle$ by single mode fiber

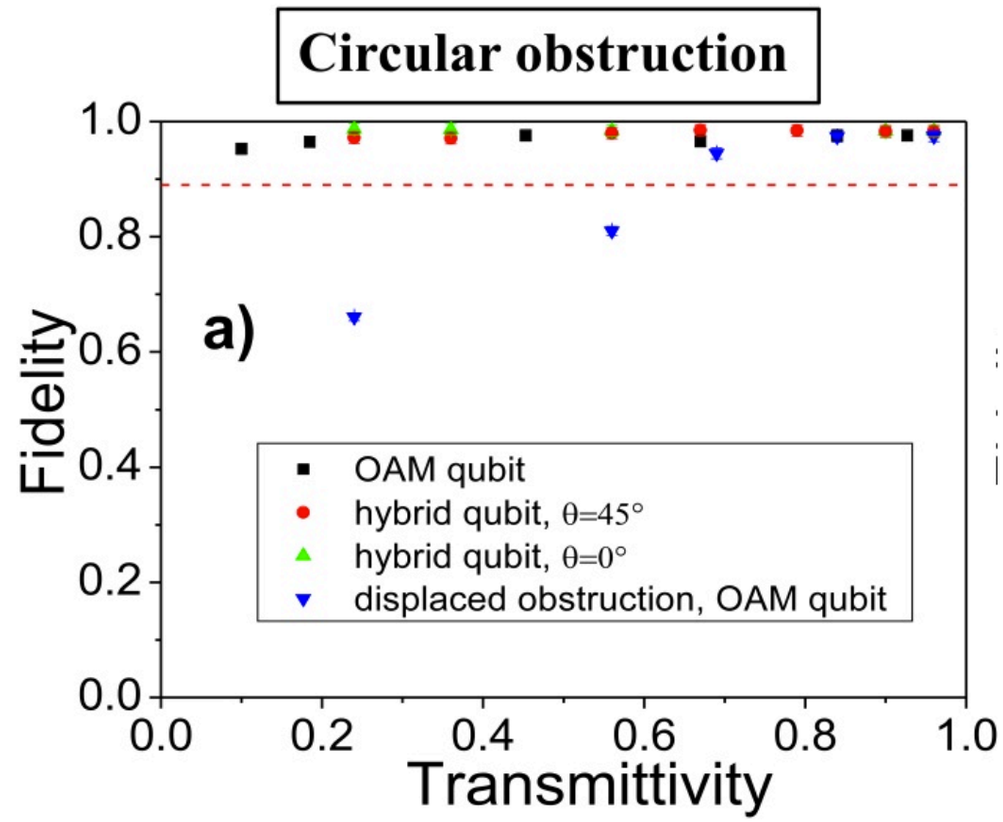
$$(C_{+1,+1}\alpha|L\rangle + C_{-1,-1}\beta|R\rangle)|0\rangle$$

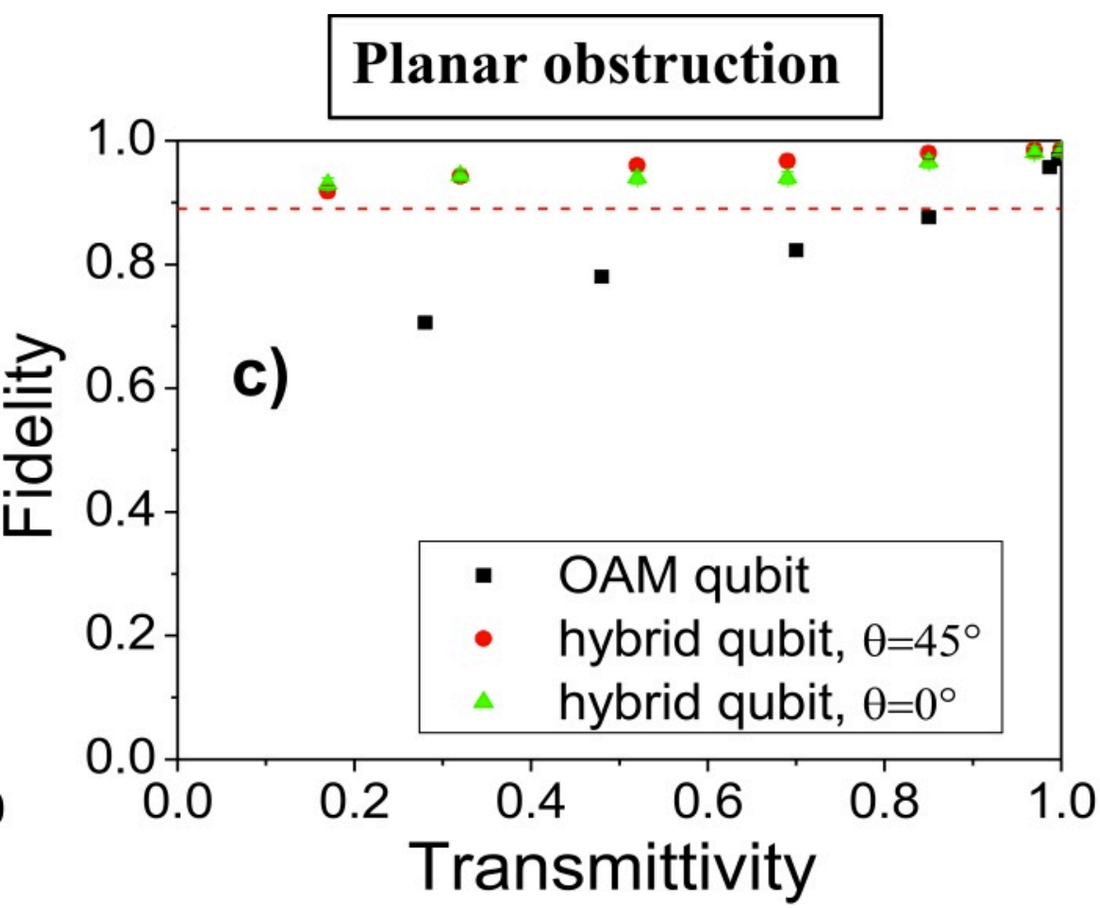
If $C_{+1,+1} = C_{-1,-1}$ $\boxed{\text{Communication fidelity is preserved.}}$

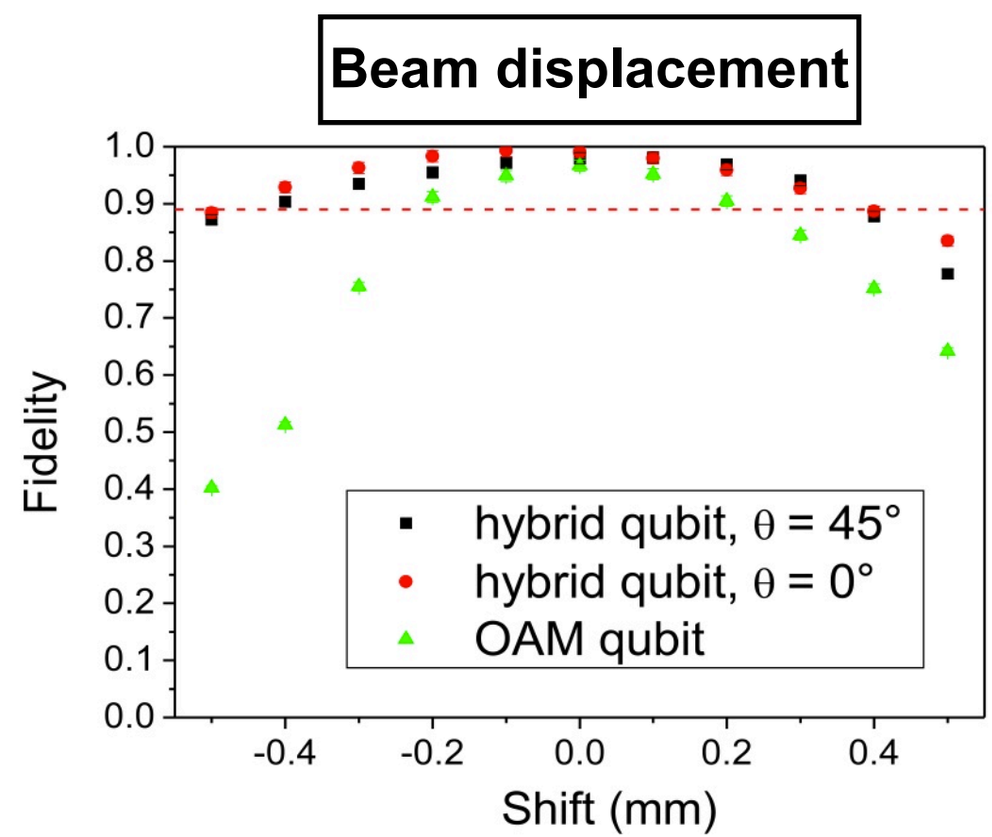
The system intrinsically discards all states outside the logical subspace.

Robustness of rotational-invariant qubits









Thank you!