Quantum Optics: when do we need it?
Part 1: for optical field

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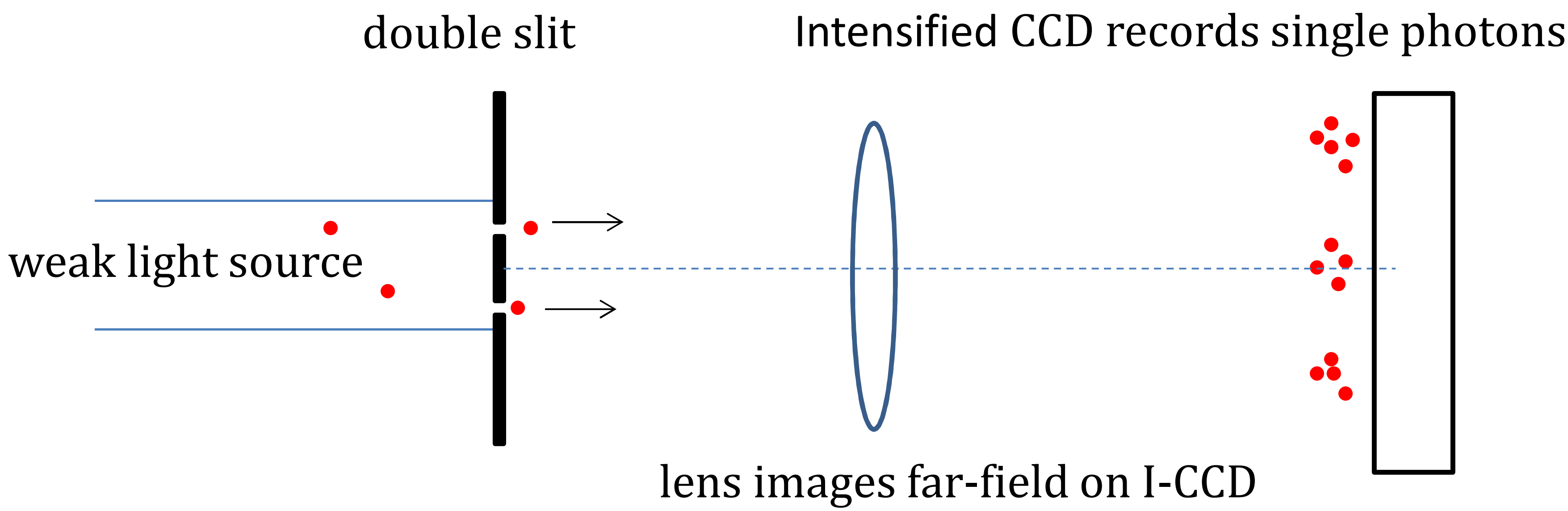
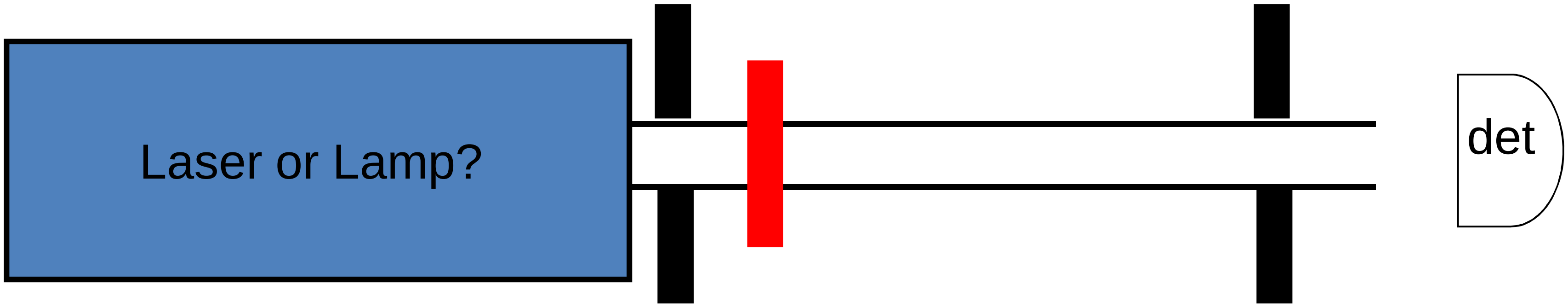
Example 1: Young's double slit with single photons



Experiment can be described without quantum optics

Example 2: How to distinguish laser from lamp light?



spatial & spectral (color) filtering

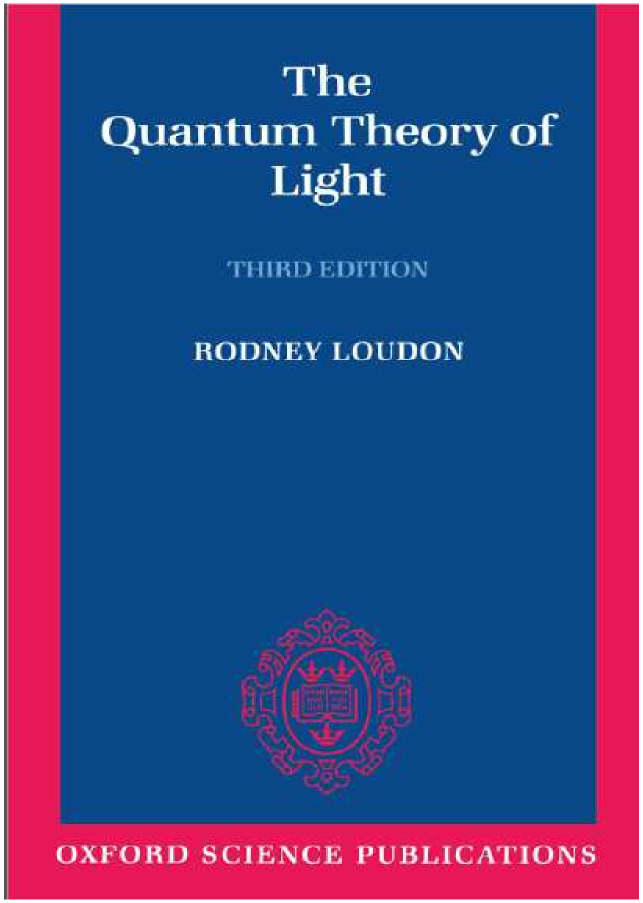
- "One-photon" optical properties (average intensity):
  - Temporal coherence & optical spectrum
  - Spatial coherence & intensity profile

Claim: "One-photon" properties don't allow you to distinguish laser/lamp

- Correlation experiments between two detected photons are needed to distinguish different quantum states of light

'Statistical Optics' by J.W. Goodman

Single-mode vs. multi-mode quantum optics



Annihilation operator: $\hat{a}$ or $\hat{a}_i$

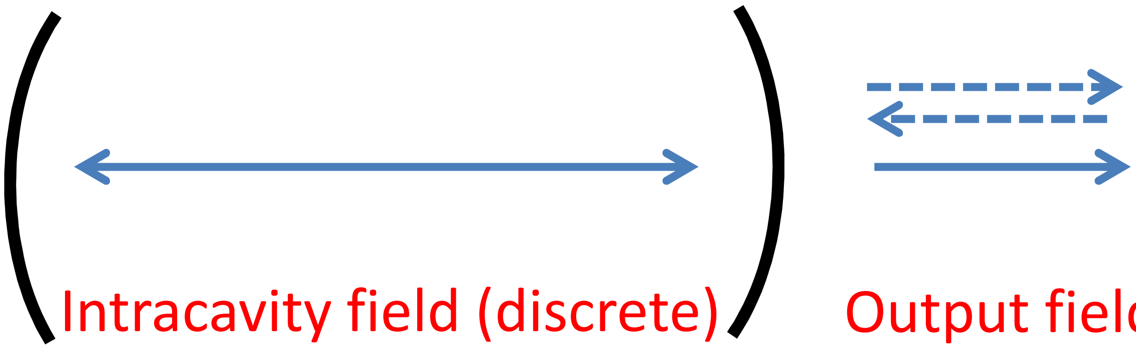
$$\left[\hat{a}_i, \hat{a}_j^{\dagger}\right] \equiv \hat{a}_i \hat{a}_j^{\dagger} - \hat{a}_j^{\dagger} \hat{a}_i = \delta_{i,j}$$

Annihilation operator: $\hat{a}(t)$ or $\hat{a}(\omega)$

$$\left(\hat{a}(\omega_1), \hat{a}(\omega_2)^{\dagger}\right) = \delta(\omega_1 - \omega_2)$$

- Single-mode = discrete mode in cavity (Chapter 5)
- Multi-mode = continuum of modes in free space (Ch. 6)

Textbooks often discuss discrete Q states of light



Intracavity field (discrete)
can be more quantum
than output field

Output field (continuous)
is affected by reflections
of vacuum field/fluctuations

Theory for "single-mode" quantum optics

- Single discrete mode: $\left[\hat{a},\hat{a}^{\dagger}\right]=\hat{a}\hat{a}^{\dagger}-\hat{a}^{\dagger}\hat{a}=1$

1. Number states:

$$\left|\psi\right\rangle=\left|n_0\right\rangle \quad\Rightarrow\quad P_n=\delta_{n,n_0}$$

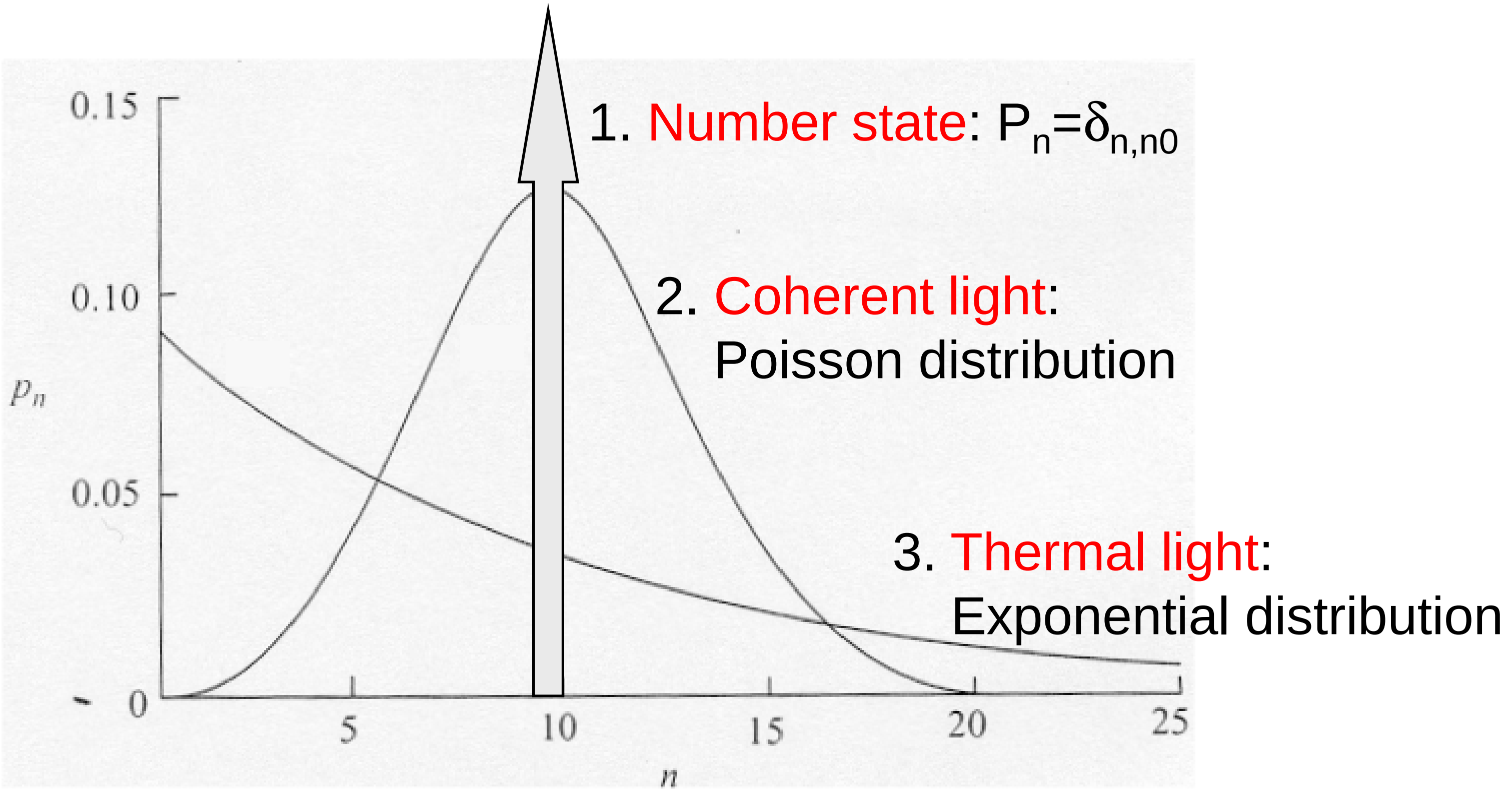
2. Coherent states:

$$\left|\psi\right\rangle=\left|\alpha\right\rangle=e^{-\left|\alpha\right|^2/2}\sum_{n=0}^{\infty}\frac{\alpha^n}{\sqrt{n!}}\left|n\right\rangle \Rightarrow P_n=e^{-\left|\alpha\right|^2}\frac{\left|\alpha\right|^{2n}}{n!} \quad \text{(Poisson distribution)}$$

3. Thermal light: $\rho=\overline{\left|\psi\right\rangle\left\langle\psi\right|}=\sum_{n=0}^{\infty}P_n\left|n\right\rangle\left\langle n\right|$

with $P_n\propto\exp\left(-n\frac{h\nu}{kT}\right)$ (Exponential distribution)

Three standard (single-mode) quantum states



"Multi-mode optics"

- Still single discrete transverse optical mode (fixed spatial structure)
- Continuous in frequency/time: $\left[\hat{a}(\omega),\hat{a}^{\dagger}(\omega')\right]=\delta(\omega-\omega')$

- Temporal coherence:

$$g^{(1)}(\tau)=\frac{\left\langle E^{-}(t)E^{+}(t+\tau)\right\rangle}{\left\langle I(t)\right\rangle}\propto\left\langle \hat{a}^{\dagger}(t)\hat{a}(t+\tau)\right\rangle=\int I(\omega)\exp(-i\omega\tau)\,d\omega$$

- Intensity correlations:

$$g^{(2)}(\tau)=\frac{\left\langle :I(t)I(t+\tau):\right\rangle}{\left\langle I(t)\right\rangle^{2}}\propto\left\langle \hat{a}^{\dagger}(t)\hat{a}^{\dagger}(t+\tau)\hat{a}(t)\hat{a}(t+\tau)\right\rangle$$

- Only for thermal light (Gaussian statistics):

$$g^{(2)}(\tau)=1+\left|g^{(1)}(\tau)\right|^{2}$$

When do you need quantum optics?

- **<u>Single-mode quantum optics</u>**
  is rarely relevant in free-space optics experiments

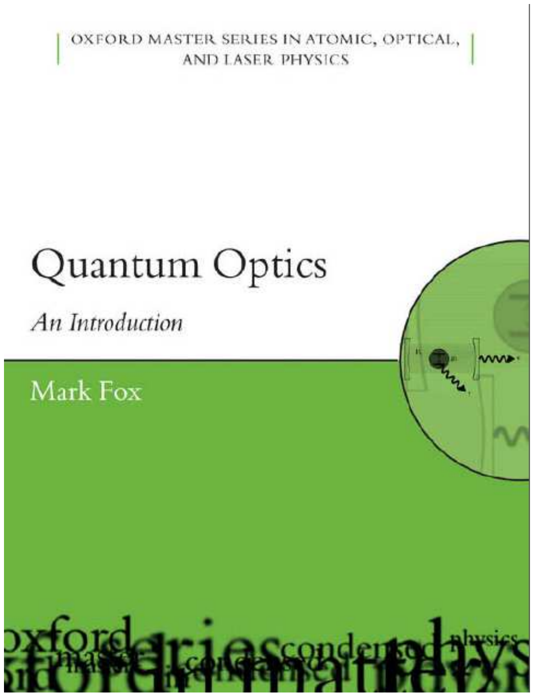
- **<u>Multi-mode quantum optics</u>** needs field quantization for:

1. Direct observation of <u>intensity noise</u> (sensitive to loss)

2. <u>Photon-photon correlations</u> (Hanbury Brown & Twiss)

2b. Interference of two single photons (Hong, Ou & Mandel)

1. Semi-classical theory of photon detection



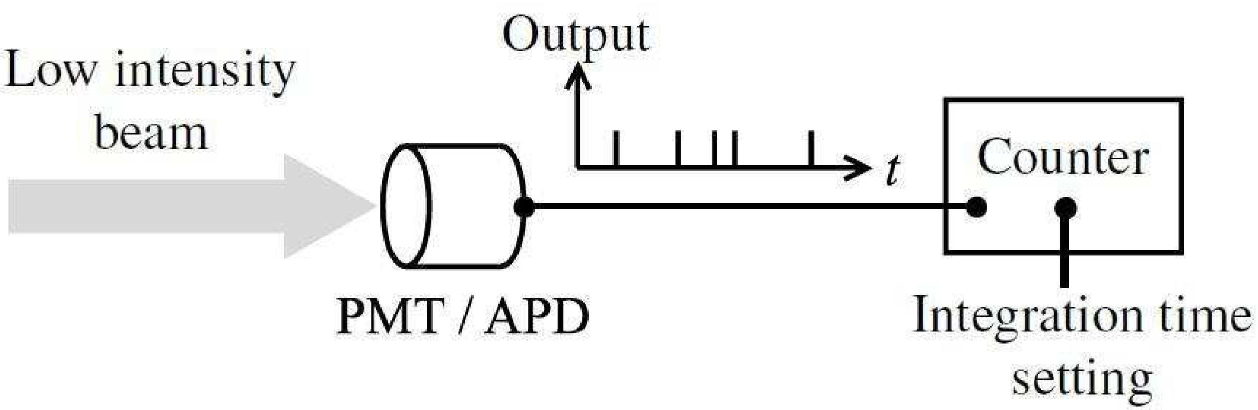
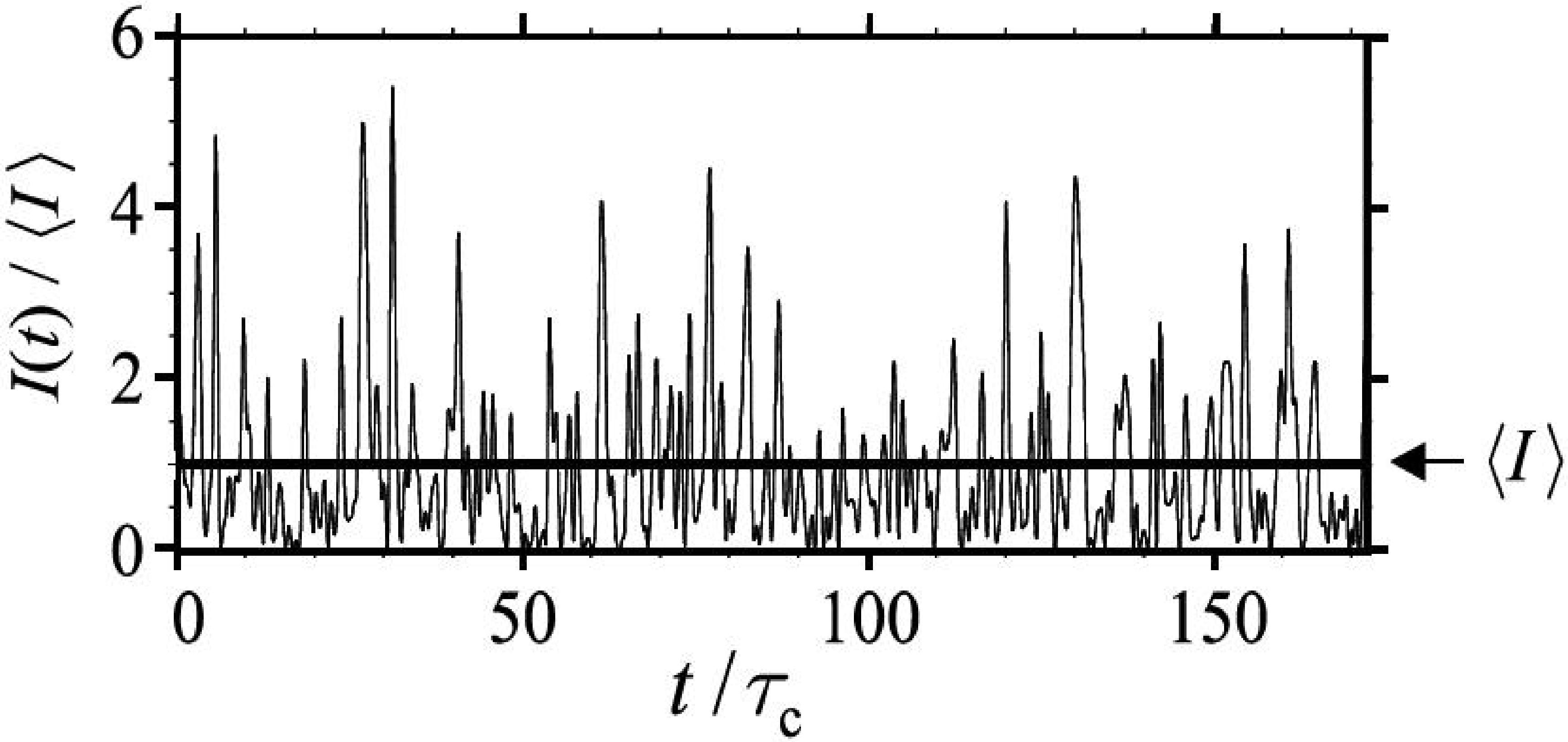


Fig. 5.1 of 'Quantum Optics' by M. Fox

- Semi-classical = treat field classically & only quantize detector

$$P(t; t + dt) = \eta . \frac{P(t)}{h\nu} . dt$$

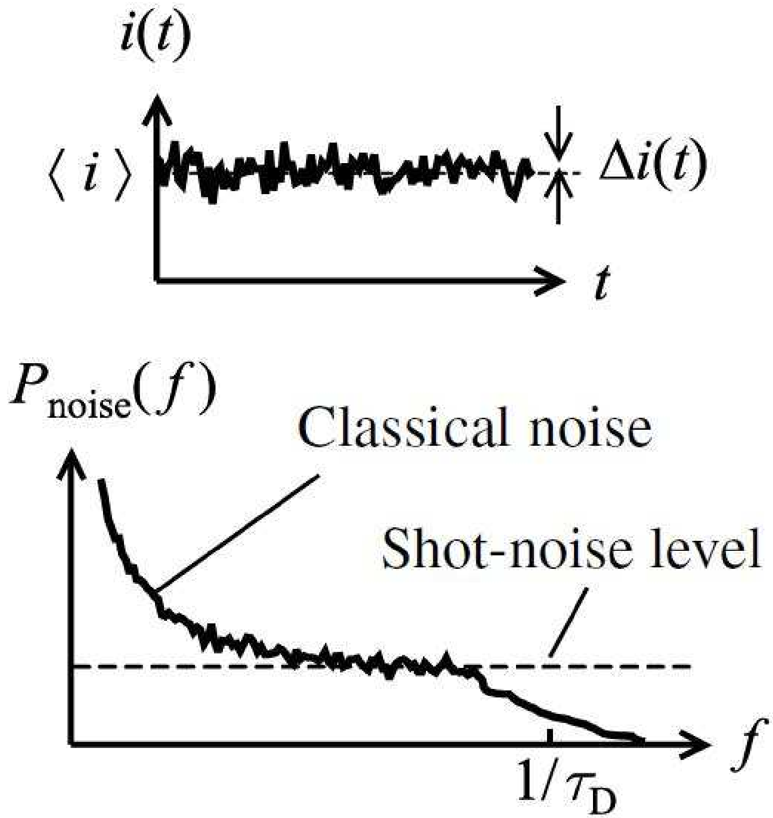
Classical description of intensity fluctuations

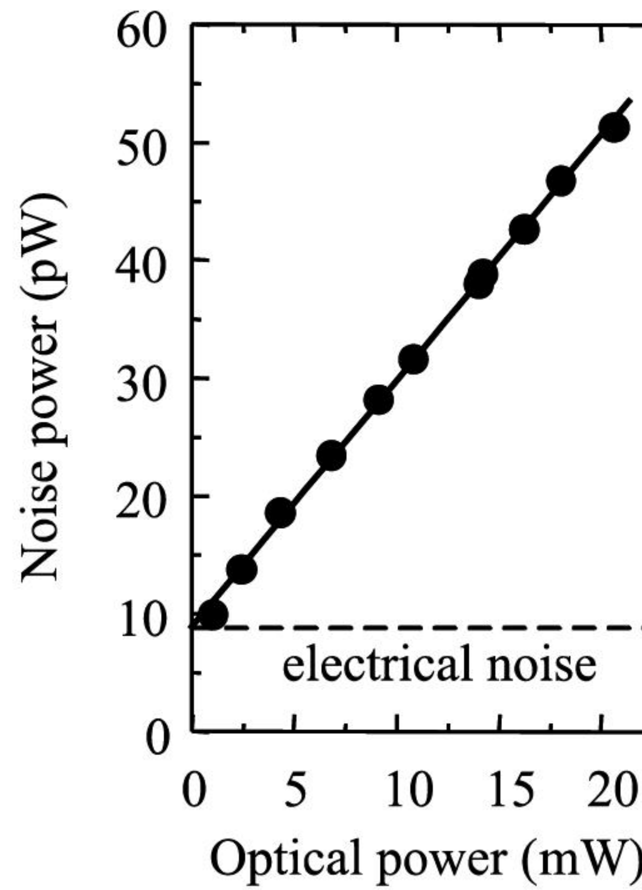


Thermal light exhibits fluctuations of both optical phase and intensity

M. Fox, Quantum Optics, Fig. 6.3

Shot noise in the detection of light

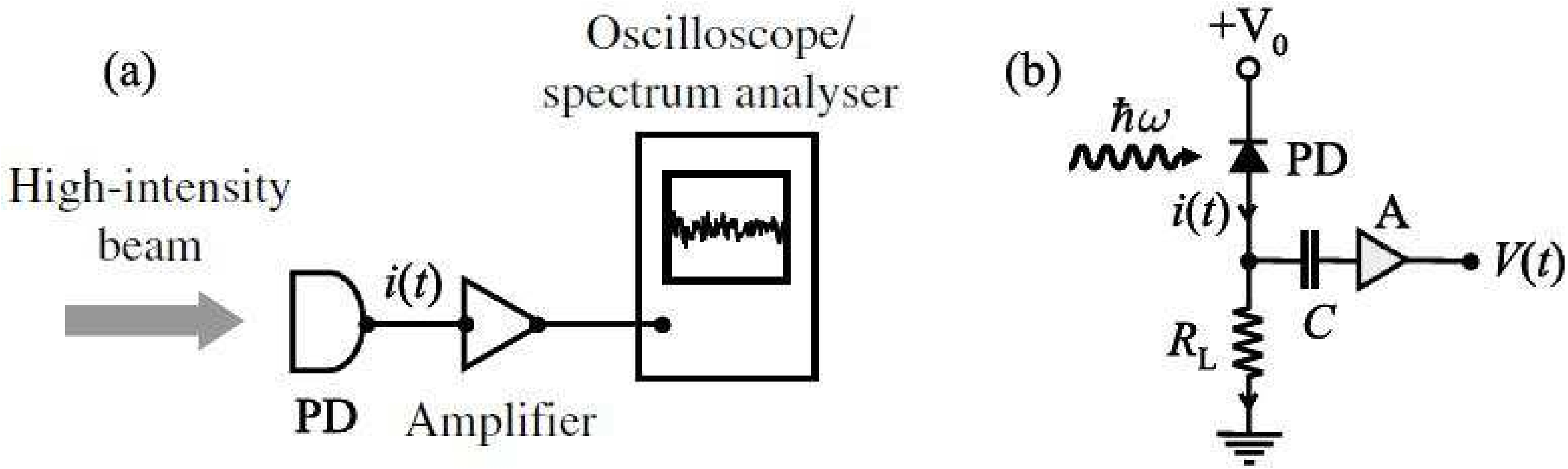




Intensity noise $I_{rms} \propto \sqrt{I_{DC}}$

M. Fox, Quantum Optics, Figs. 5.10 & 5.12

Shot noise can only dominate when V_{det} > 50 mV



$$V_{electric\ noise} = \sqrt{4kTR_L.\Delta f}$$

(≈ 0.9 nV/√Hz in 50Ω at roomtemperature)

$$V_{shot\ noise} = R_L.\sqrt{2qi.\Delta f}$$ (scales as √N)

$$V_{shot\ noise} > V_{electric\ noise} \Rightarrow V_{det} = R_L i > 2kT/q \quad 50\ mV\ @\ 300K$$

Sub-Poissonian light

Franck-Hertz experiment: sub-shot noise intensity fluctuations

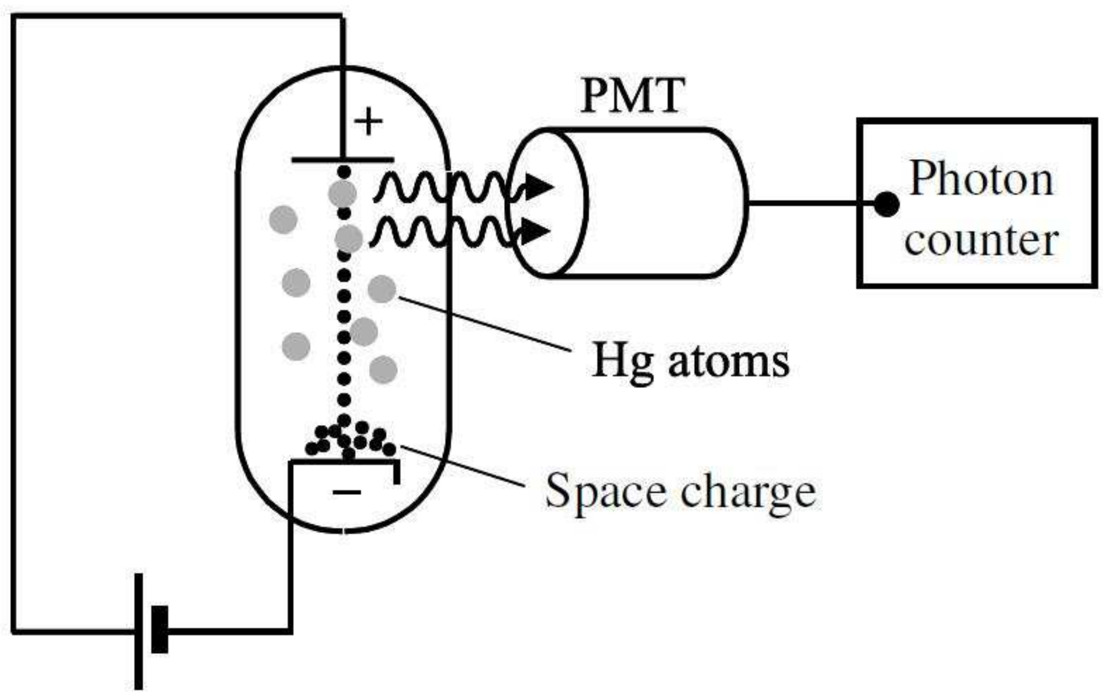
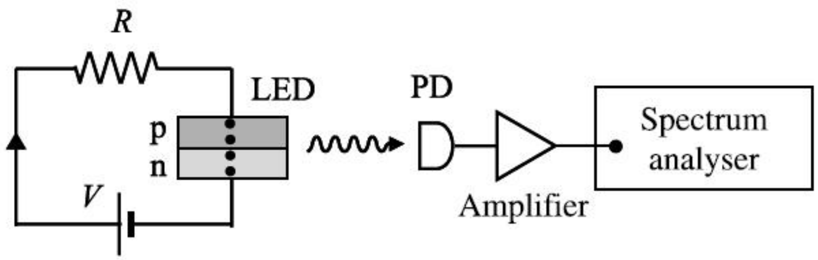


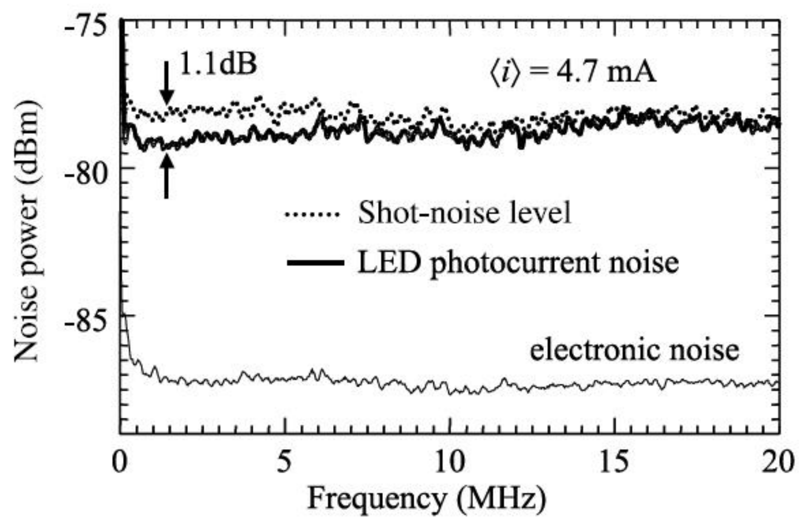
Fig. 5.14 of 'Quantum Optics' by M. Fox

Sub-Poissonian light

Electric current doesn't suffer from shot-noise

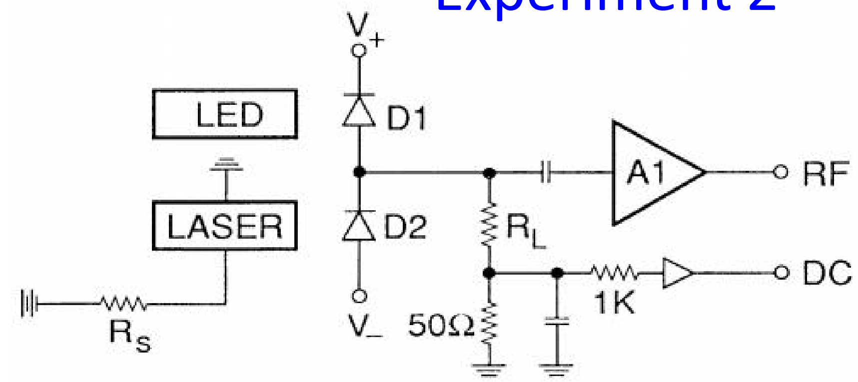
Experiment 1

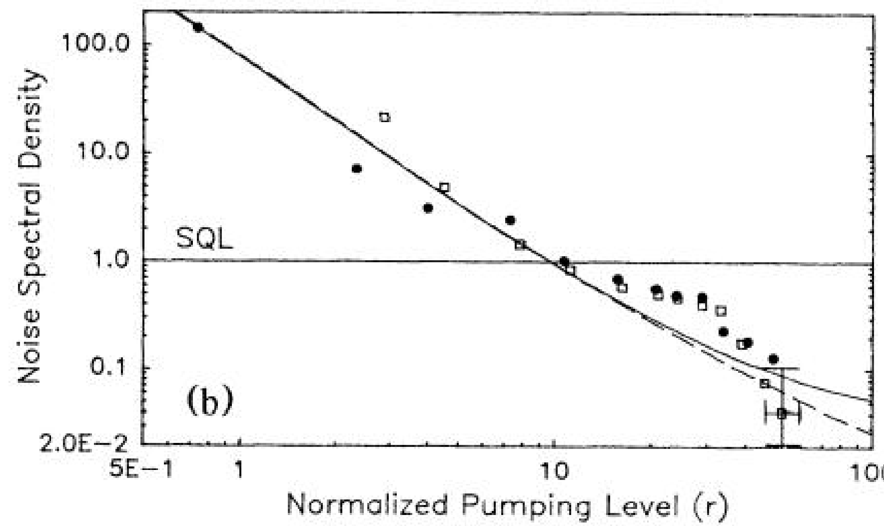




M. Fox, Quantum Optics, Fig. 5.15

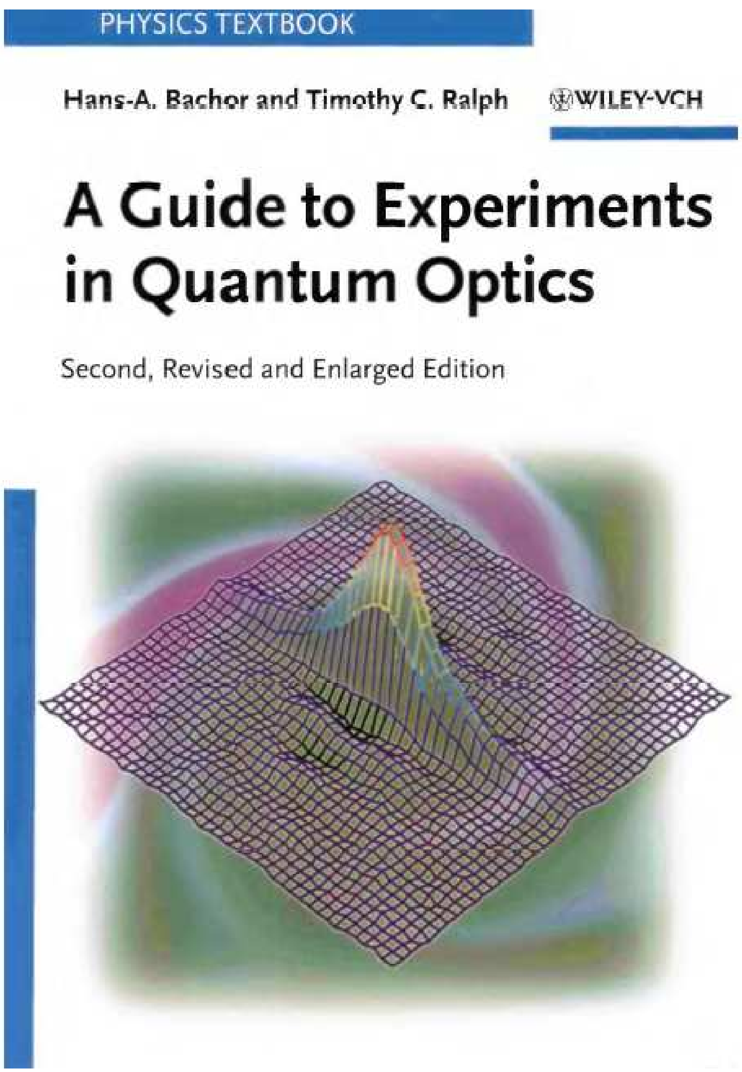
Experiment 2

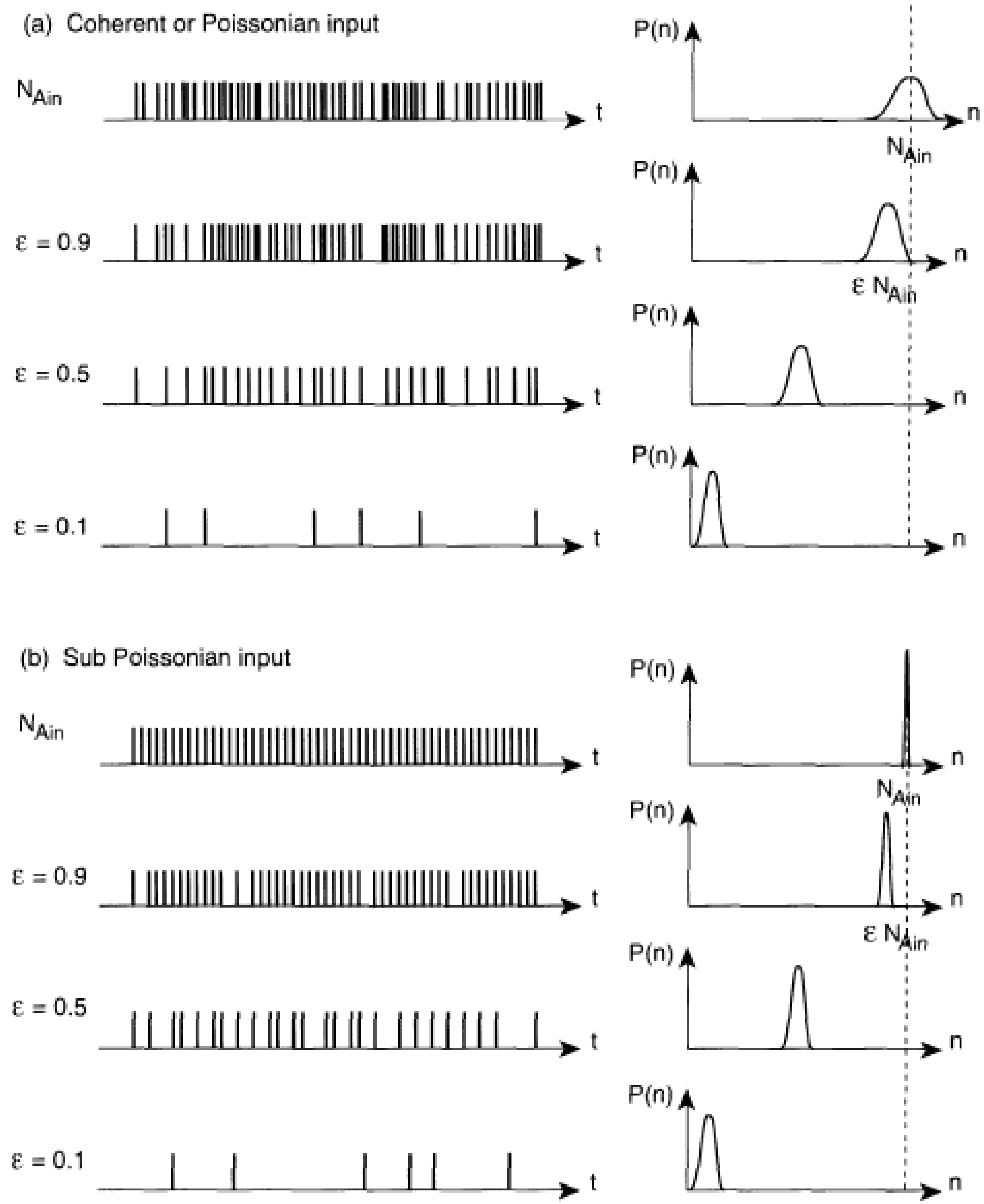




W.H. Richardson et al., PRL 66,2867 (1992)

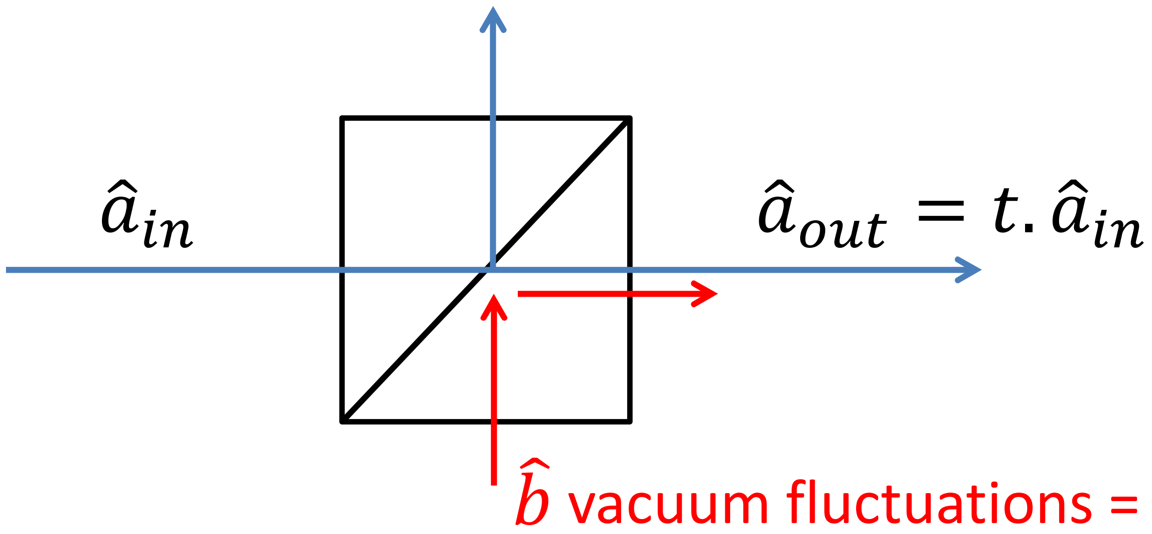
Optical loss kills the sub-Poissonian character





Optical loss kills the sub-noise character

Optical loss introduces quantum noise, as it acts like beamsplitter



$$\left[\hat{a}_{out}, \hat{a}_{out}{}^{\dagger}\right] = |t|^2.\left[\hat{a}_{in}, \hat{a}_{in}{}^{\dagger}\right] = |t|^2 \,??$$

$\hat{a}_{out}$=t. $\hat{a}_{out} + r.\hat{b}$ with $[\hat{b}, \hat{b}^{\dagger}] = 1$

$$\left[\hat{a}_{out}, \hat{a}_{out}{}^{\dagger}\right] = |t|^2 + |r|^2 = 1$$

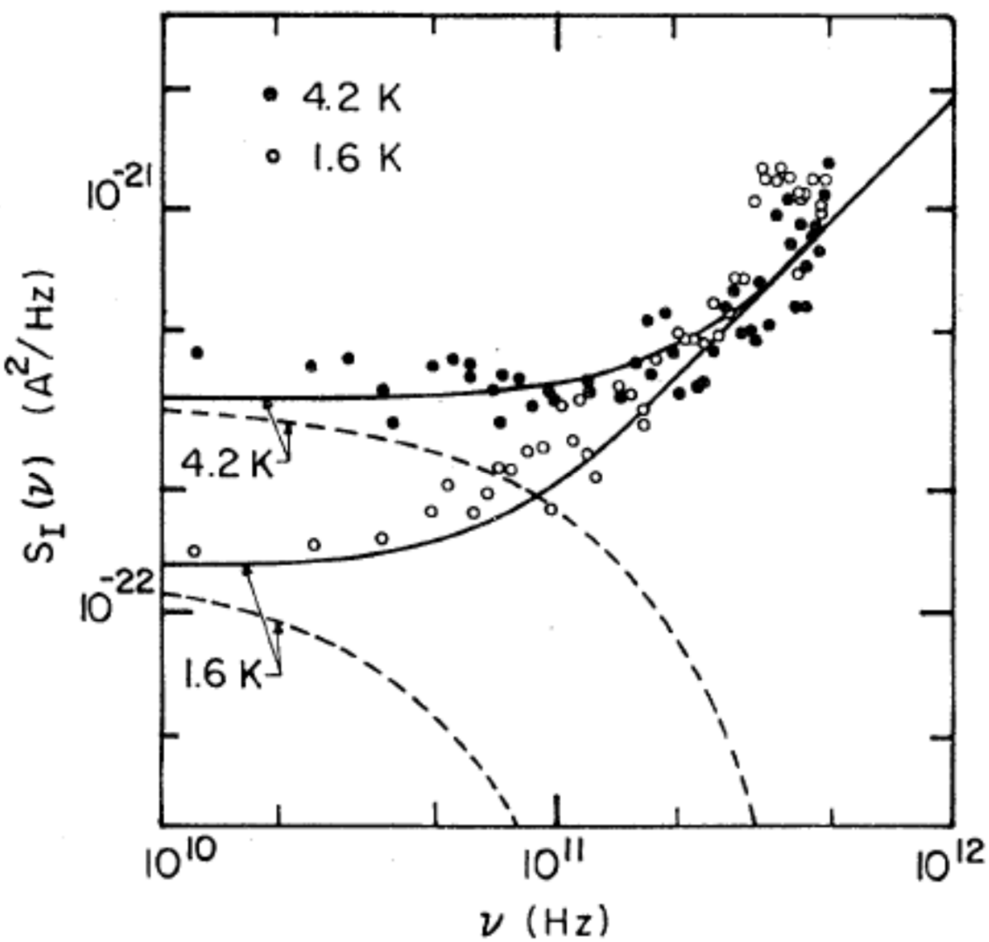
Quantum noise = Vacuum fluctuations

- Strength of the vacuum fluctuations depends on measurement:

$$\langle 0|\hat{a}^\dagger\hat{a}|0\rangle = 0 \qquad \langle 0|\tfrac{1}{2}\left(\hat{a}^\dagger\hat{a}+\hat{a}\hat{a}^\dagger\right)|0\rangle = \tfrac{1}{2} \qquad \langle 0|\hat{a}\hat{a}^\dagger|0\rangle = 1$$

- Vacuum fluctuations (in single transverse mode) :
  - 1 photon/second / per unit spectral bandwidth = 0.12 μW /nm @ 800 nm

Quantum noise can be measured



Experiment: current noise in Josephson tunnel junction, mixed down to detection frequencies

$$\left(\hat{a}^\dagger\hat{a}+\hat{a}\hat{a}^\dagger\right)-\text{form}$$

$$S_I(\nu)=\frac{4h\nu}{R}\left(\frac{1}{\exp\left(h\nu/kT\right)-1}+\frac{1}{2}\right)$$

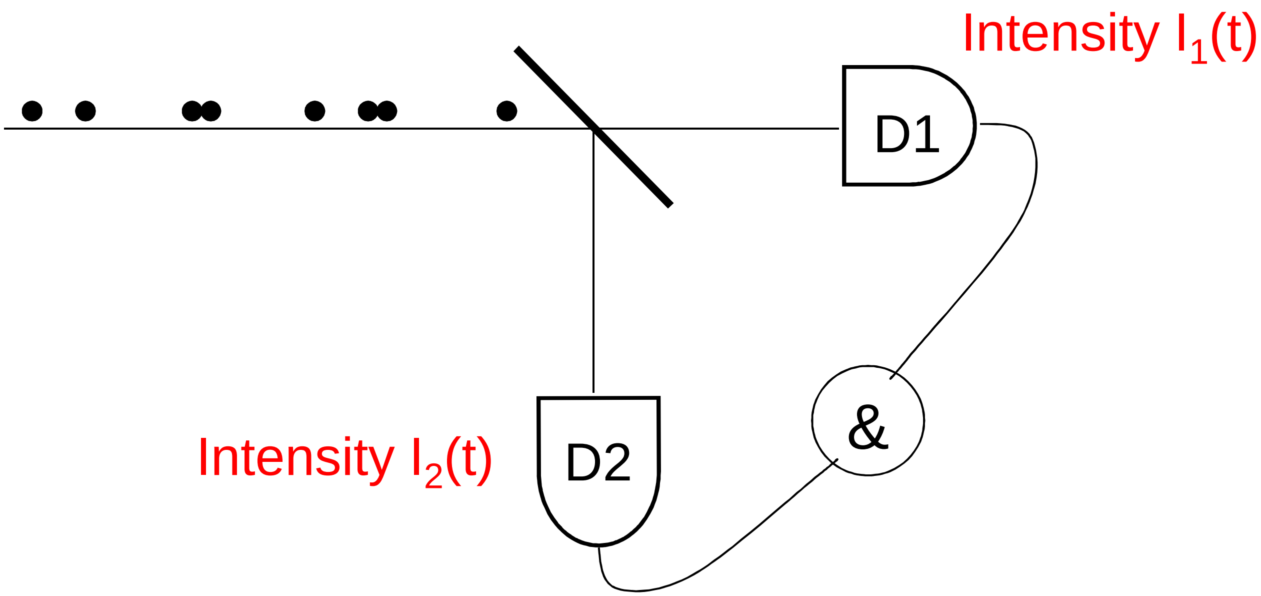
Limit kT >> hν $$S_I(\nu)=\frac{4kT}{R}$$

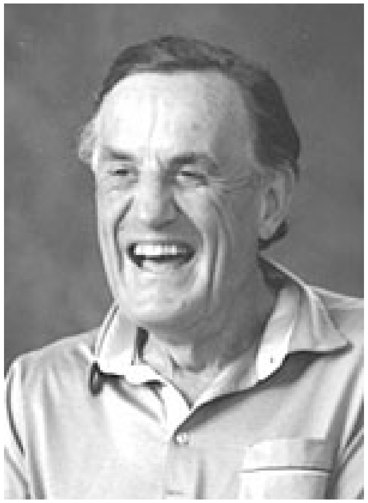
Limit kT << hν: 1/2 noise quantum per "time/frequency mode"

R.H. Koch et al., PRB 26, 74 (1982)
Introduction in book C.W. Gardiner, 'Quantum Noise'

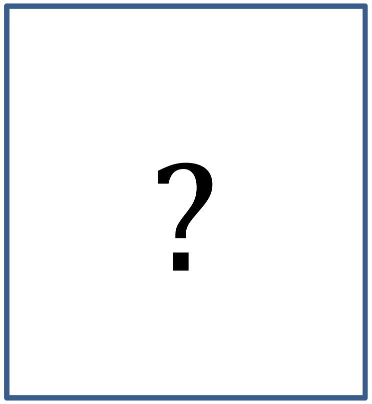
2. Intensity fluctuations & photon correlations

Hanbury Brown & Twiss experiment





Hanbury Brown

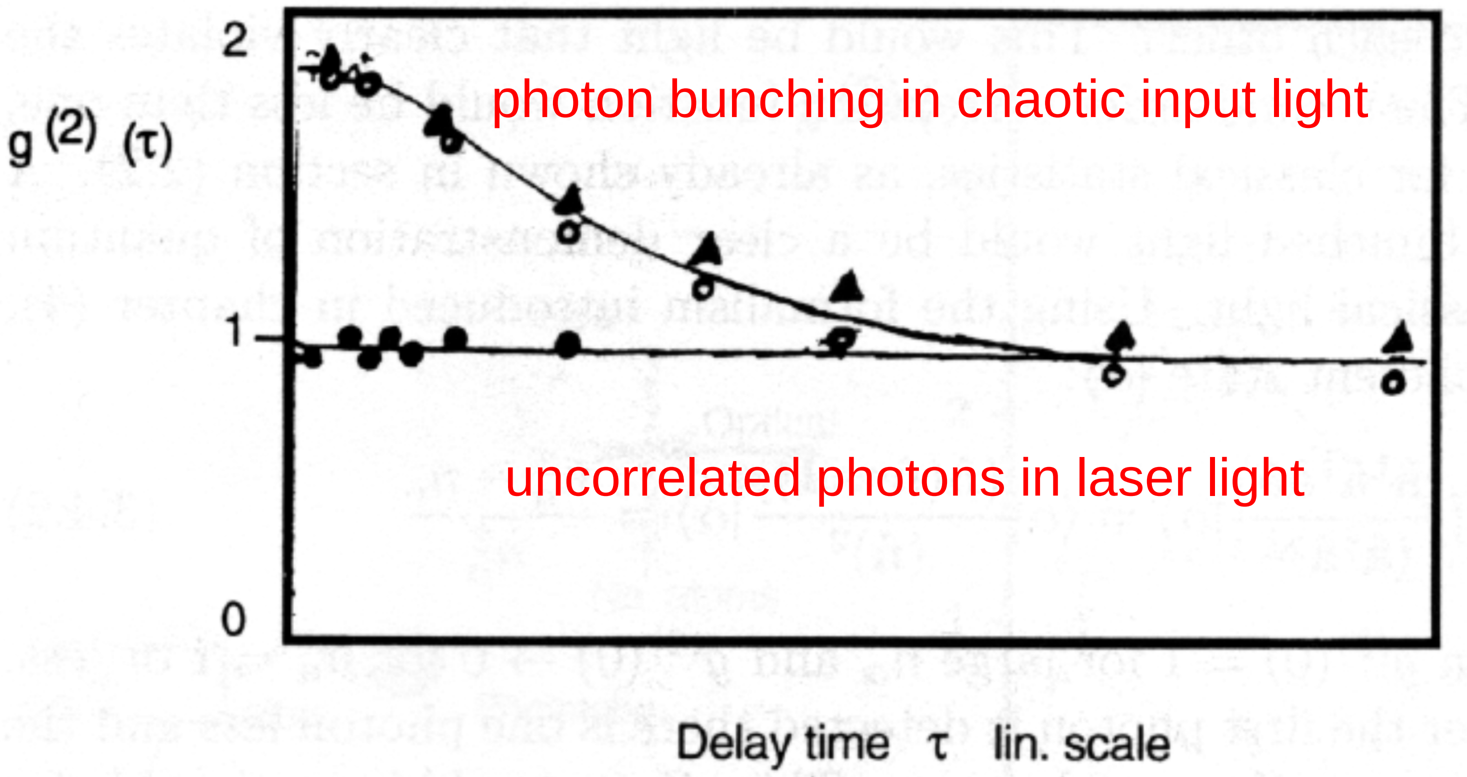


Twiss

Second-order (intensity-intensity) correlation function

$$g^{(2)}(\tau) \equiv \frac{\langle I(t+\tau) I(t) \rangle}{\langle I(t) \rangle^2}$$

Intensity fluctuations distinguish laser from chaotic light

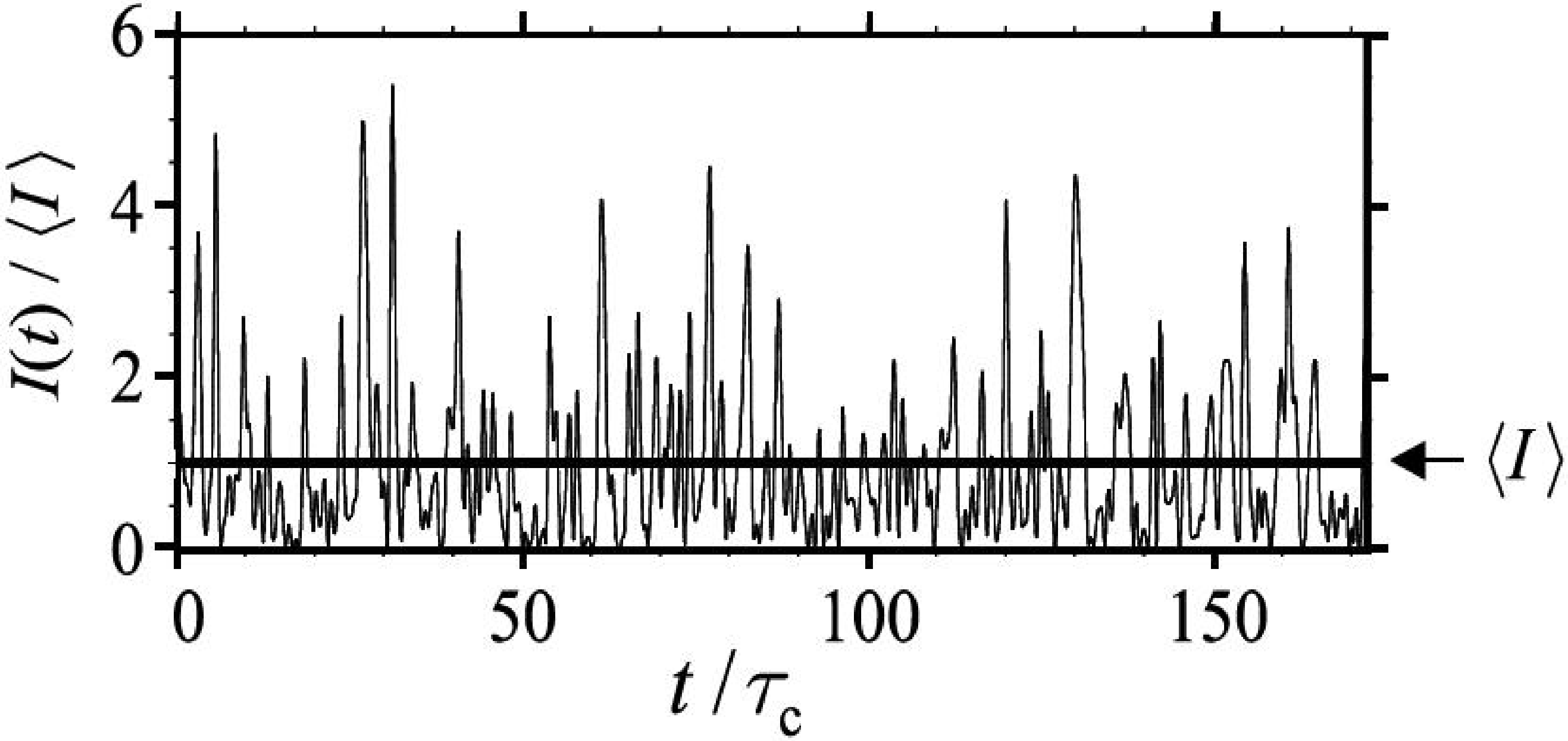


- *Solid dots = laser above threshold*
- *other symbols = chaotic light*

F.T. Arecchi et al., PRL17, 260 (1966)

$$g^{(2)}(\tau) \equiv \frac{\langle I(t+\tau)\, I(t)\rangle}{\langle I(t)\rangle^2}$$

Classical description of intensity fluctuations

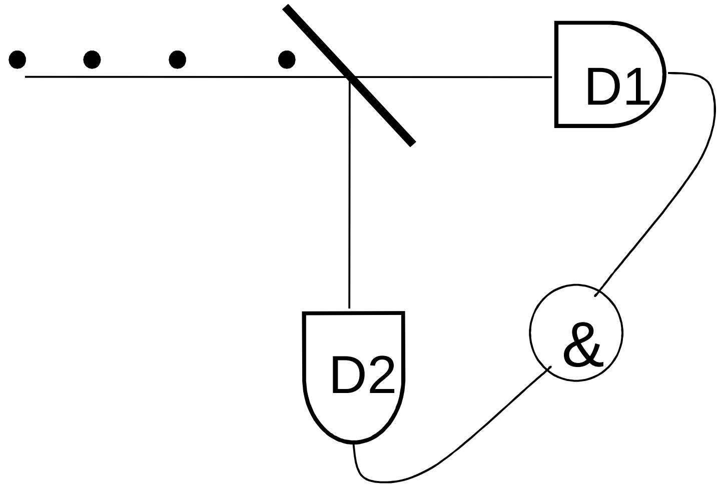


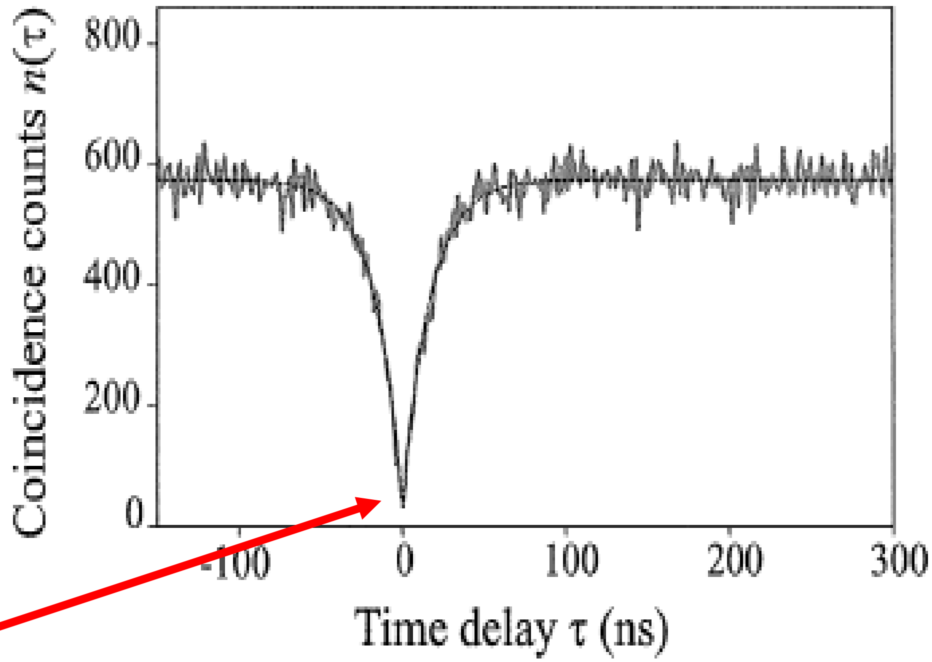
Thermal light exhibits fluctuations of both optical phase and intensity

M. Fox, Fig. 6.3

Anti-bunching in fluorescence single CdSe quantum dot

Anti-bunching observed for light from single-photon sources

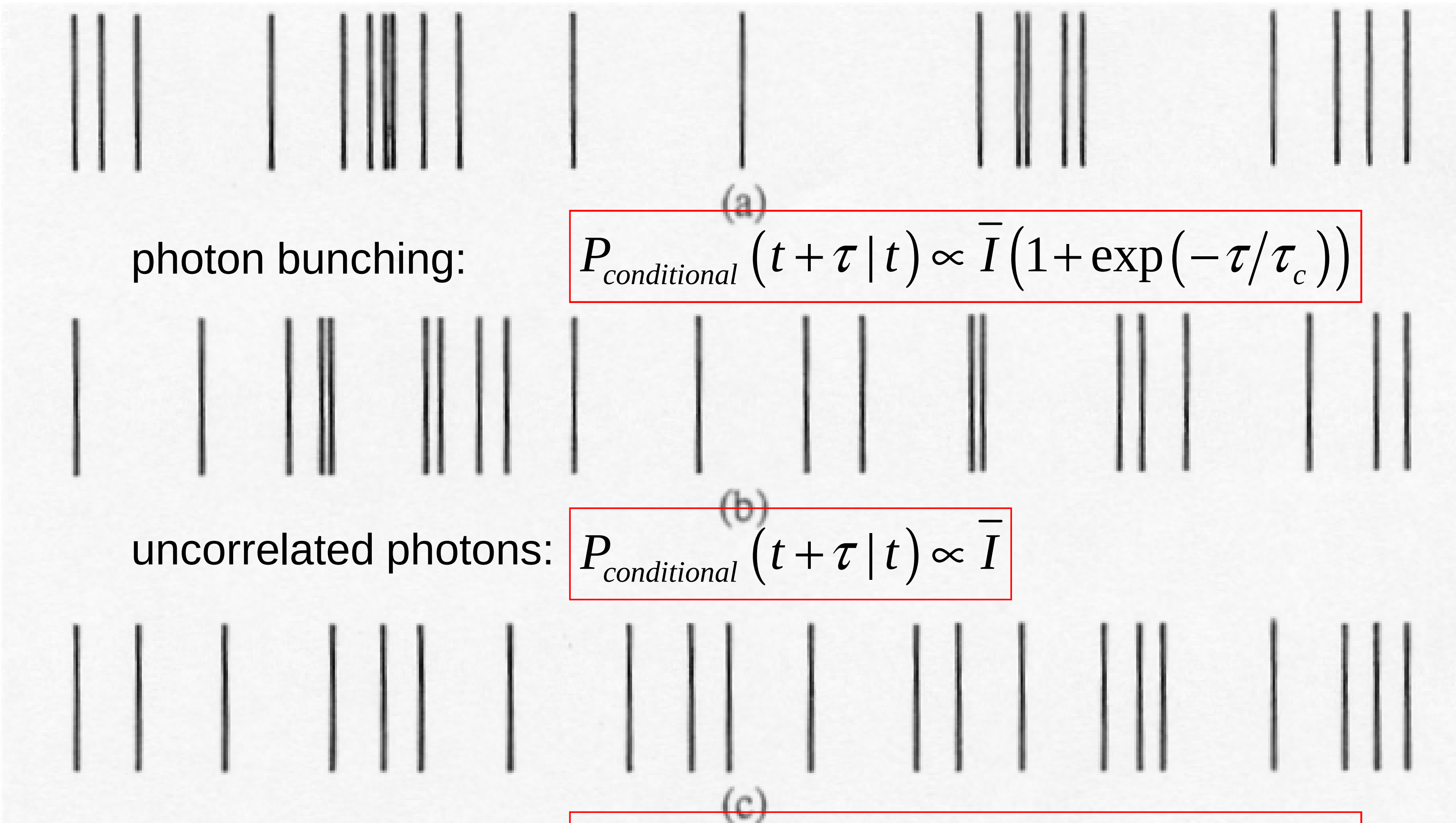




Anti-bunching at input: never two photons together!

G. Messin et al., Opt. Lett. 26, 1891 (2001)

Time-series of photon detection events

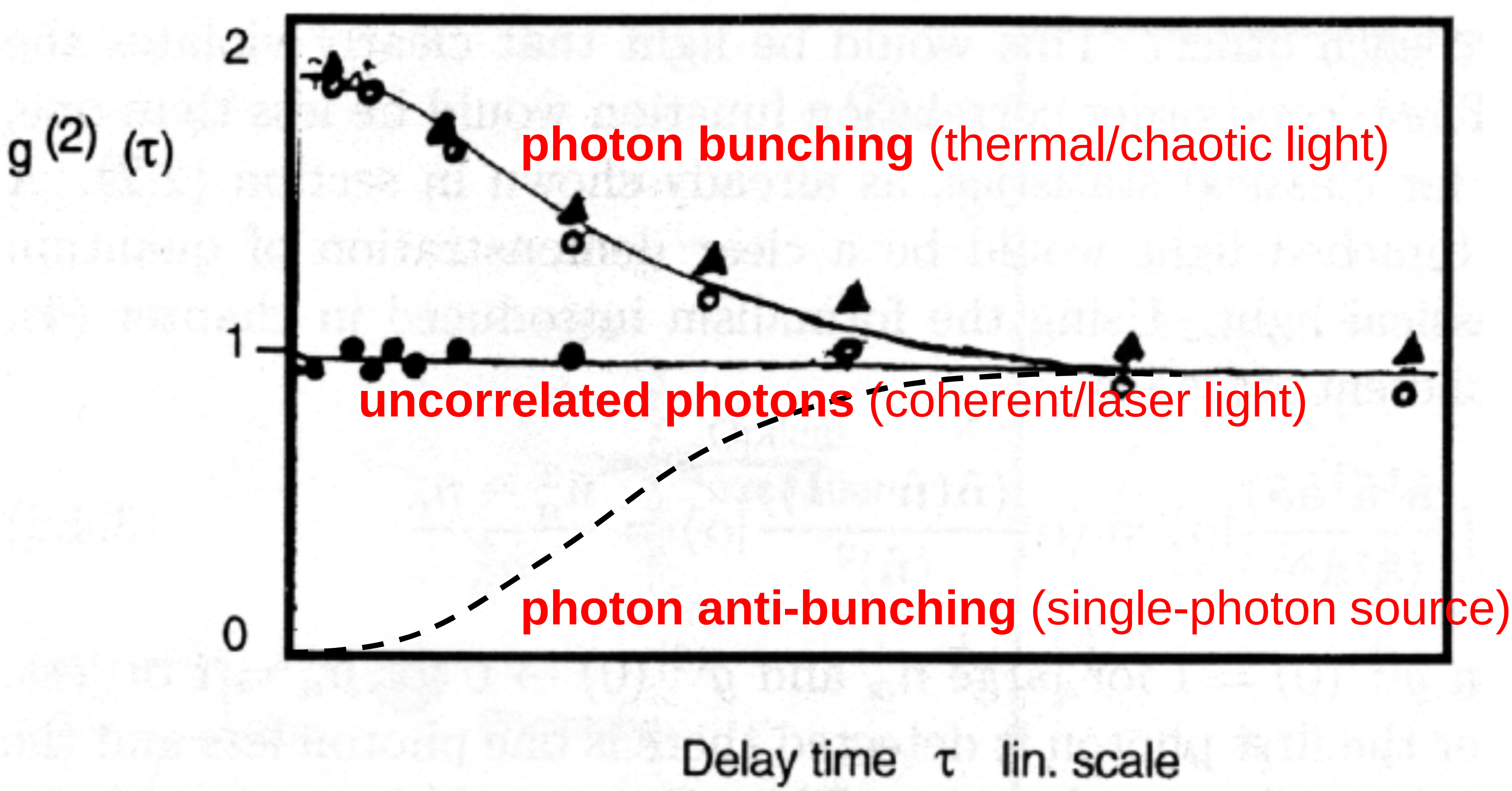


photon bunching: $P_{conditional}\left(t+\tau\,|\,t\right) \propto \bar{I}\left(1+\exp\left(-\tau/\tau_c\right)\right)$

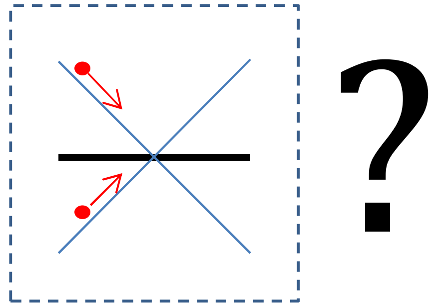
uncorrelated photons: $P_{conditional}\left(t+\tau\,|\,t\right) \propto \bar{I}$

photon anti-bunching: $P_{conditional}\left(t+\tau\,|\,t\right) \propto \bar{I}\left(1-\exp\left(-\tau/\tau_c\right)\right)$

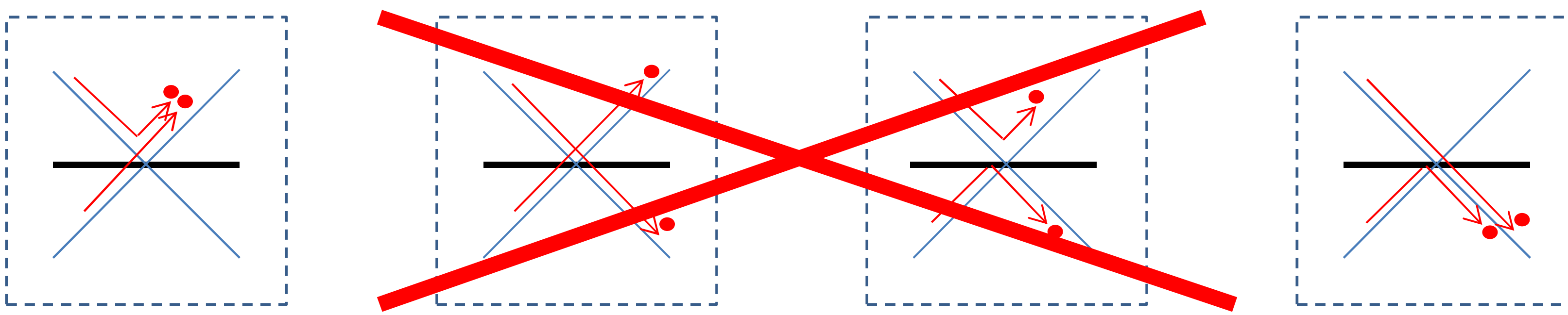
Intensity correlations for three quantum states of light



3. Interference between two identical single photons

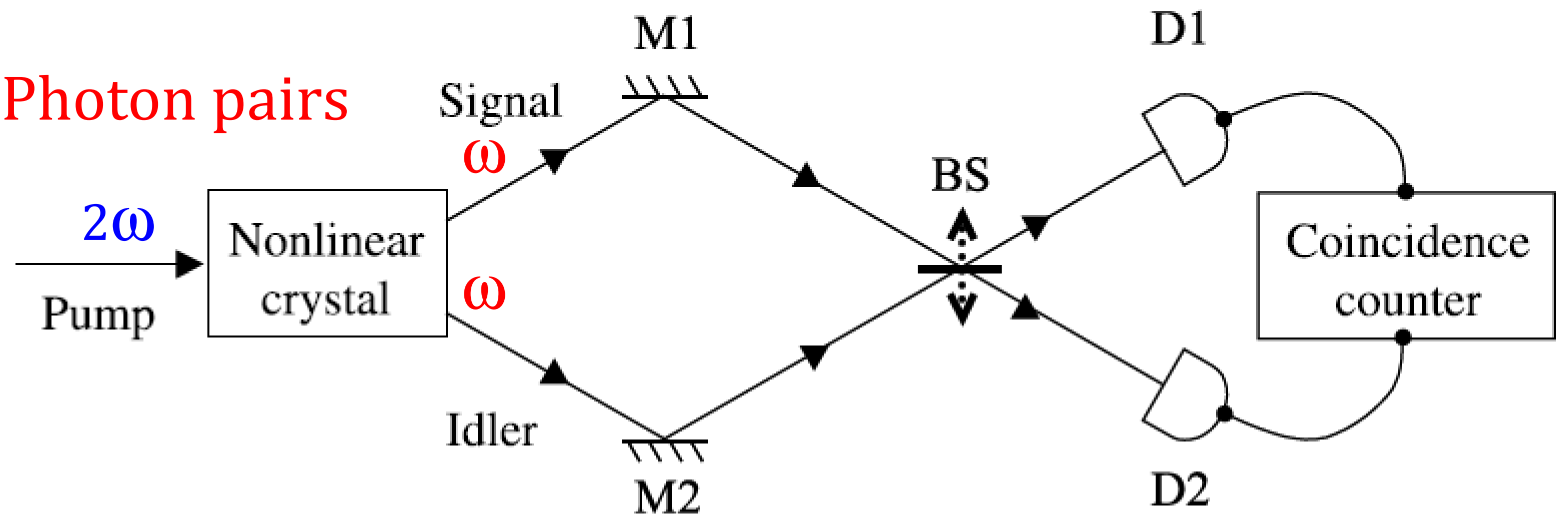


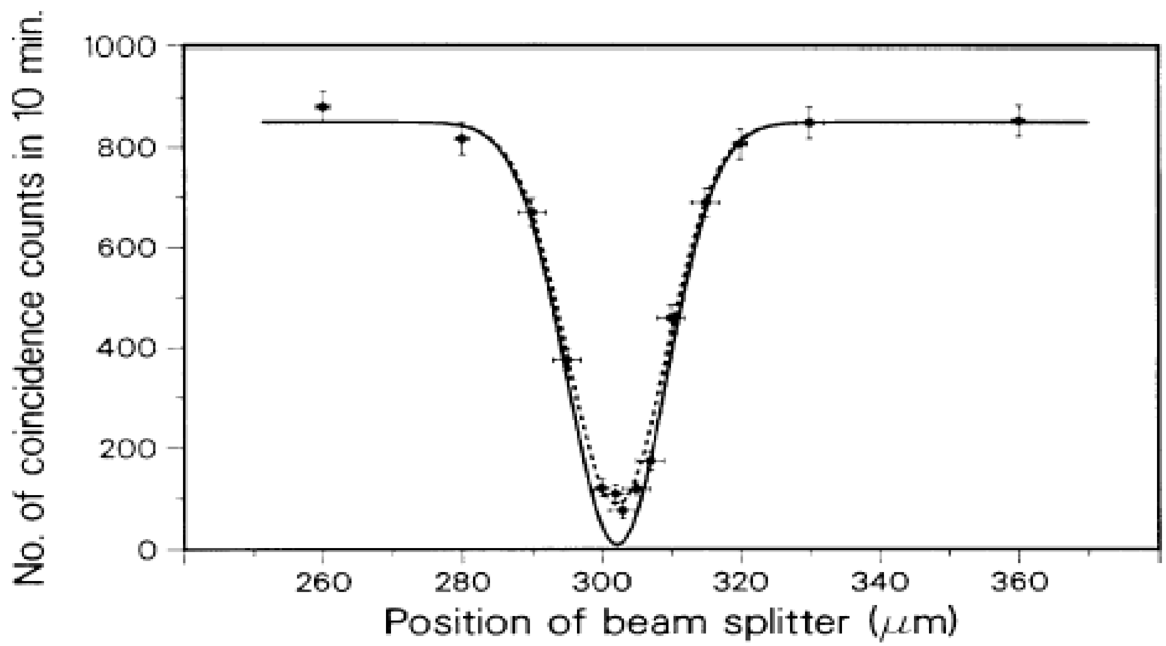
Four possible scenarios



Quantum prediction: no coincidences for identical photons

Photon bunching in Hong-Ou Mandel experiment

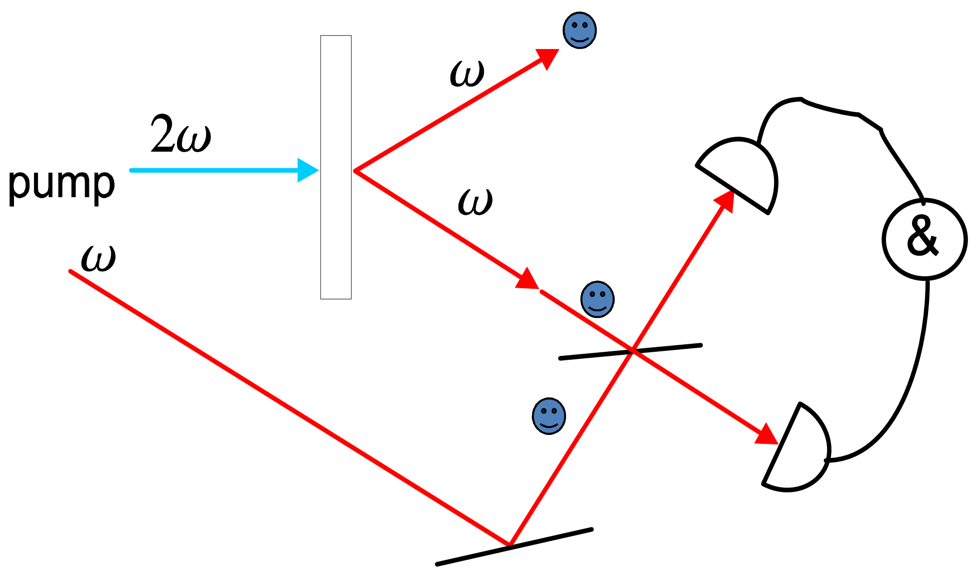


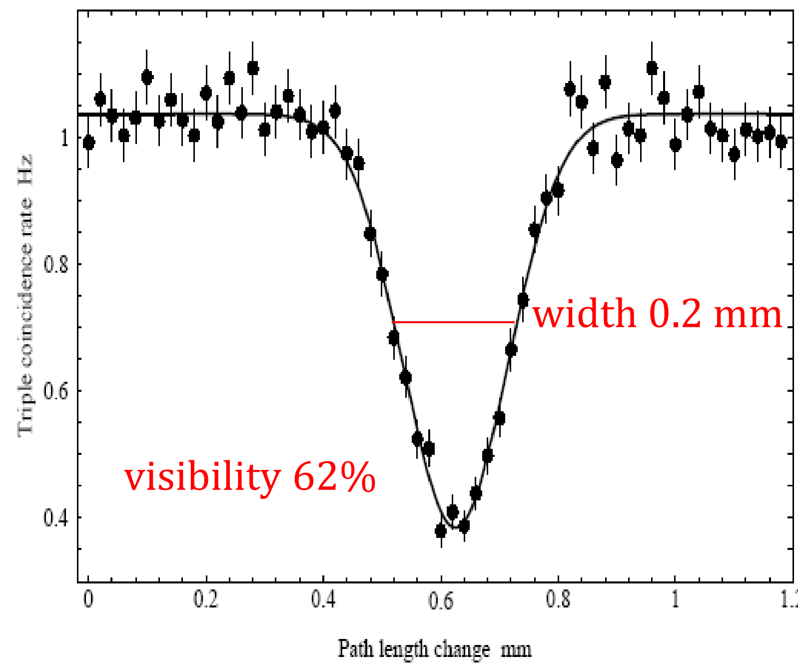


No fringes!

Hong, Ou, Mandel, PRL59, 2044 (1987)

Interference between two 'independent' photons



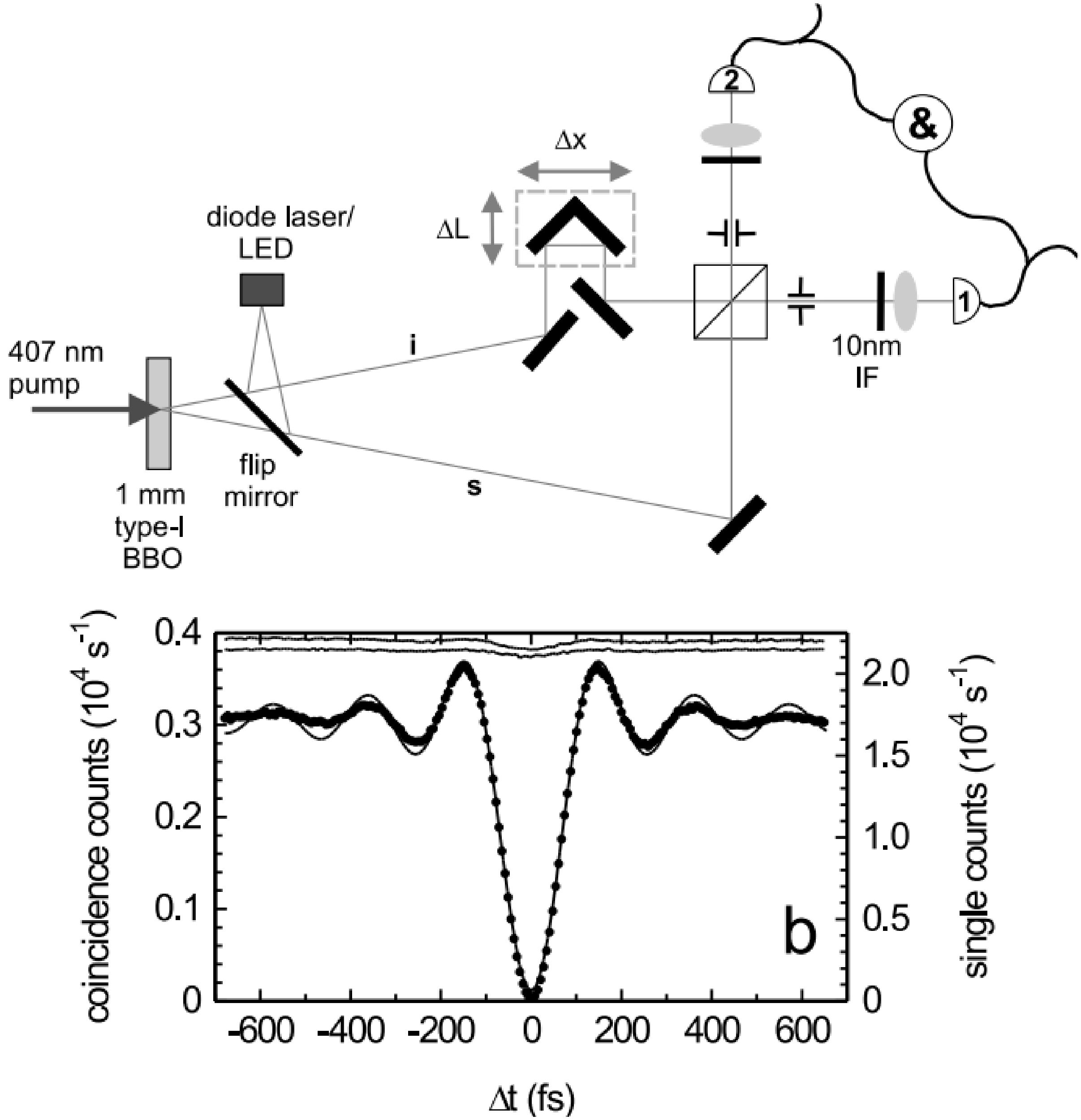


Identical photons:

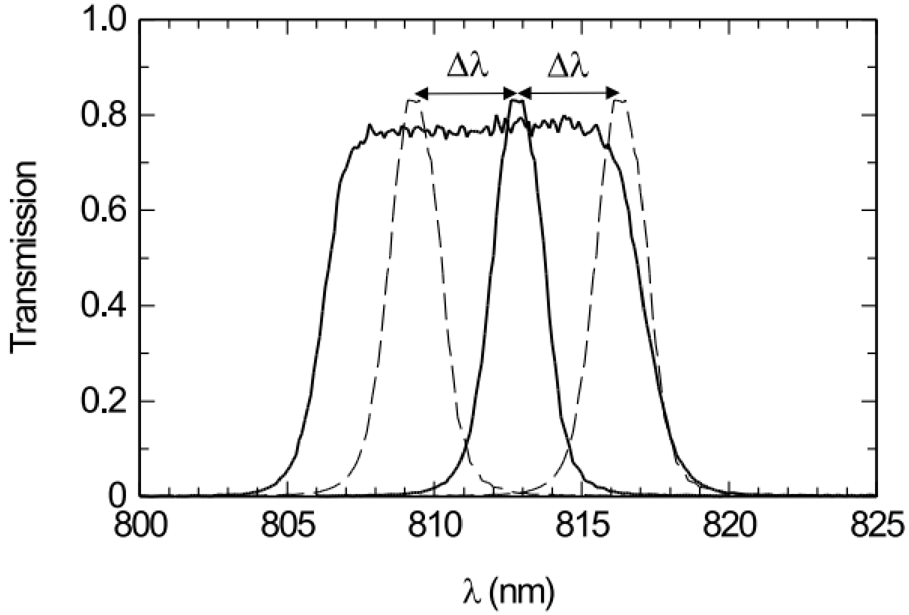
- Same spatial profile (project on single-mode fiber)
- Same time/frequency profile (use pulsed light)

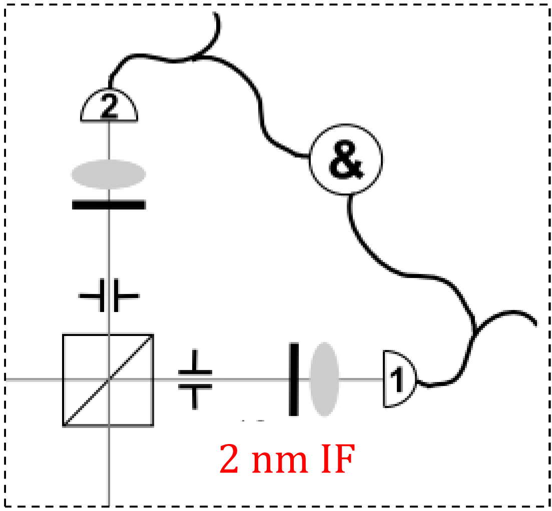
Rarity, Tapster & Loudon (1997)

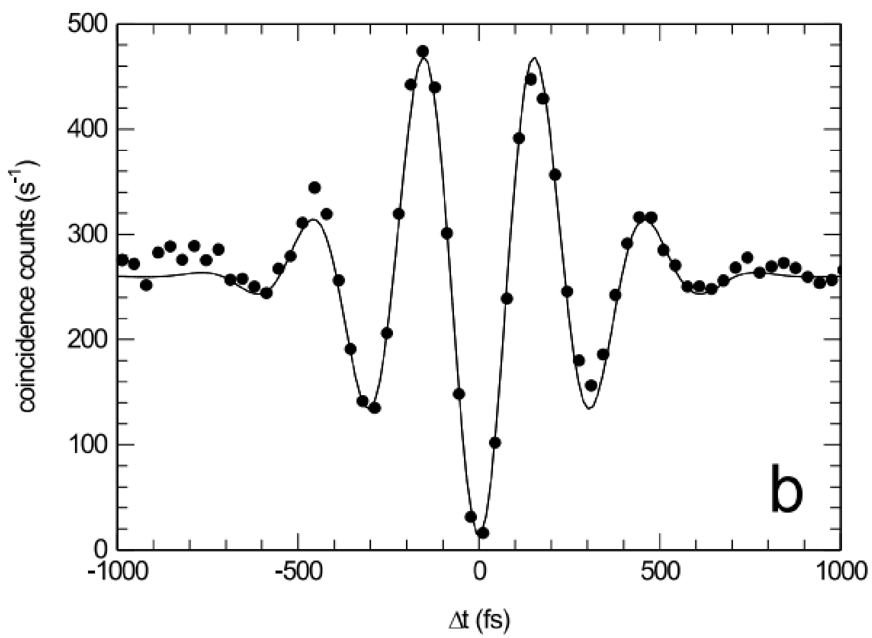
Our results with HOM interference



Our results with HOM interference







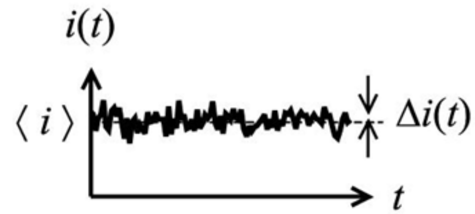
Conclusions: When do you need quantum optics?

- **<u>Single-mode quantum optics</u>**

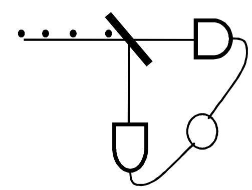
  is rarely relevant in free-space optics experiments

- **<u>Multi-mode quantum optics</u>** needs field quantization for:

1. Direct observation of <u>intensity noise</u> (difficult; sensitive to loss)



2. <u>Photon-photon correlations</u> (Hanbury Brown & Twiss)



2b. Interference of two single photons (Hong, Ou & Mandel)