



European Laboratory for  
Non-Linear Spectroscopy



Università di Firenze  
Dipartimento di Fisica



INO-CNR  
ISTITUTO  
NAZIONALE DI  
OTTICA



REGIONE  
TOSCANA



# Manipulation of quantum light at the single-photon level and by ultrafast pulse-shaping techniques

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# Outline

## Part 1

Manipulating CV quantum states at the single-photon level and quantum homodyne tomography characterization

- Single-photon addition and subtraction
- Sequences and superpositions of quantum operators
- Direct probing of fundamental quantum rules
- Noiseless amplification

## Part 2

Investigating the mode structure of ultrashort pulsed quantum light states

- Adaptive homodyne detection
- Spectral-temporal shaping of quantum states

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# Photon creation and annihilation operators

Creation and annihilation operators:

$$\hat{a}^\dagger |n\rangle = \sqrt{n+1} |n+1\rangle$$

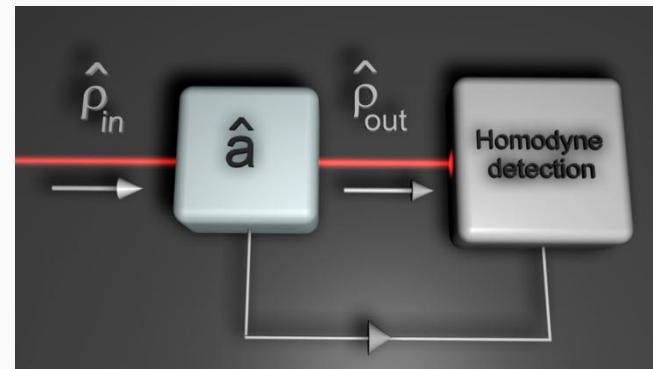
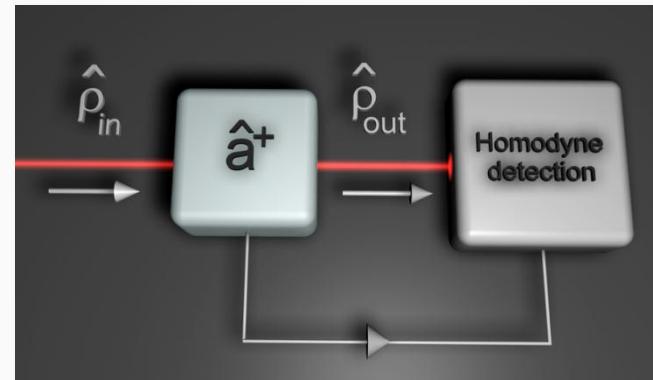
$$\hat{a} |n\rangle = \sqrt{n} |n-1\rangle$$

$$\hat{a}^\dagger \hat{\rho} \hat{a}$$

“Photon-added” state

$$\hat{a} \hat{\rho} \hat{a}^\dagger$$

“Photon-subtracted” state



Conditional generation schemes

# Adding a single photon to a state of light

Parametric amplification  
in a nonlinear crystal

In the low-gain regime       $g \equiv \chi t \ll 1$   
(eliminates higher-order excitations)

$$H = i\hbar\chi(\hat{a}_s^\dagger\hat{a}_i^\dagger - \hat{a}_s\hat{a}_i)$$

$$|\psi(t)\rangle \approx [1 + g(\hat{a}_s^\dagger\hat{a}_i^\dagger - \hat{a}_s\hat{a}_i)] |\psi(0)\rangle$$

Inject a seed pure state in the signal mode

$$|\psi(0)\rangle \equiv |\psi\rangle_s |0\rangle_i$$

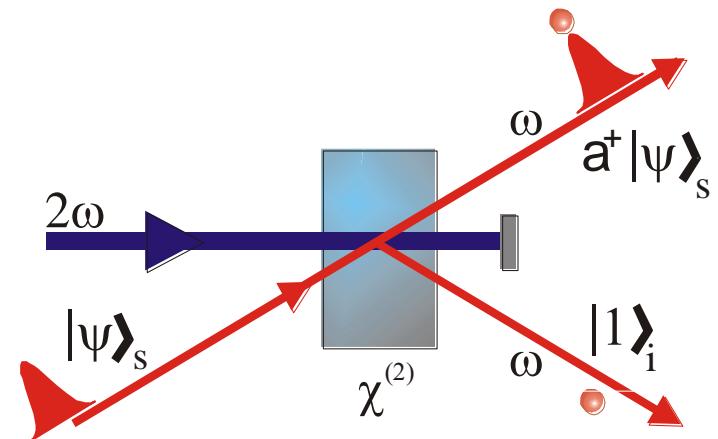


$$|\psi\rangle_{out} = |\psi\rangle_s |0\rangle_i + g\hat{a}_s^\dagger |\psi\rangle_s |1\rangle_i$$

Stimulated emission regime

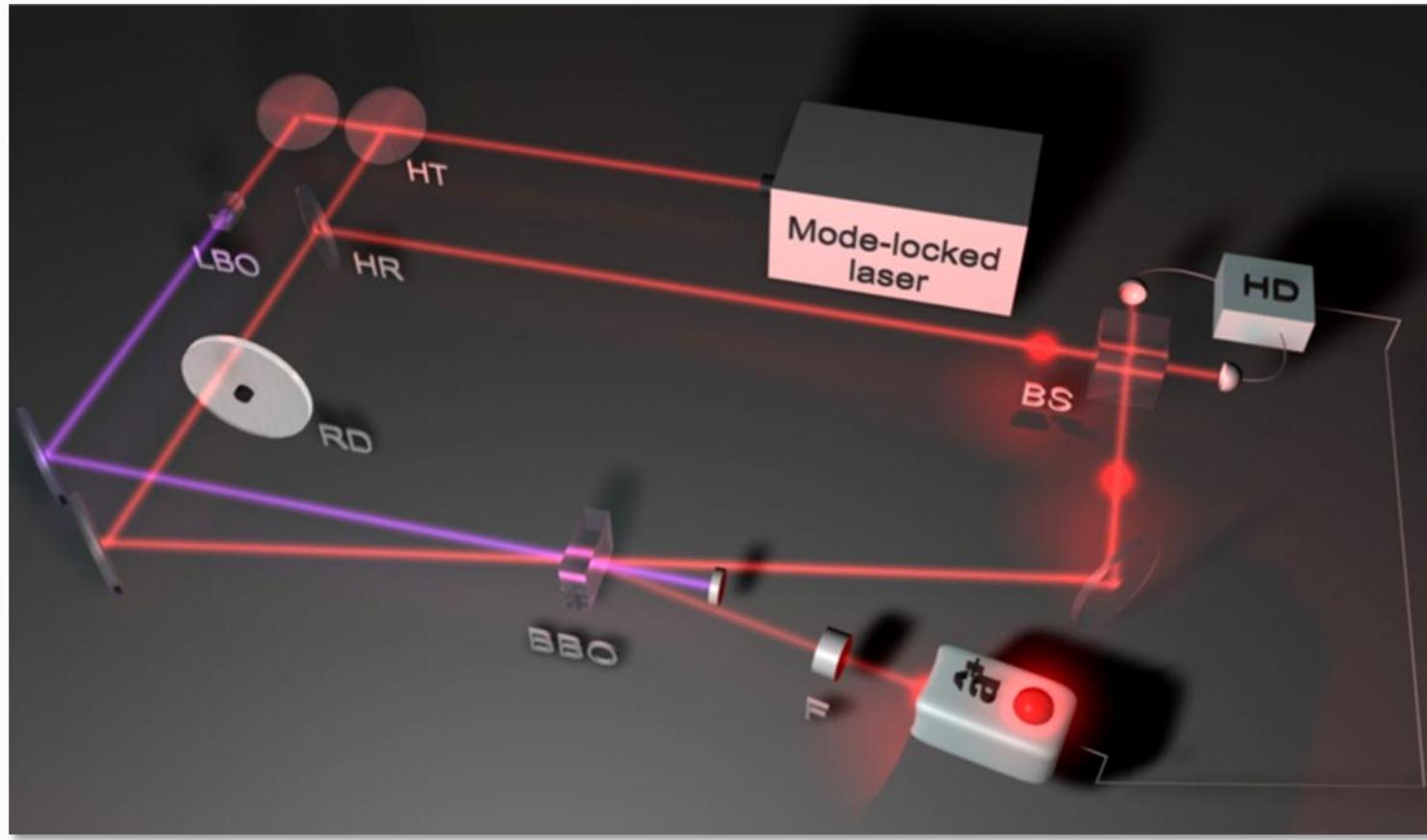
Emission probability increases with  $\langle n \rangle$

$$\begin{aligned} p_{st} &\propto |g|^2 \langle \psi | \hat{a}\hat{a}^\dagger | \psi \rangle = \\ &= |g|^2 (1 + \langle \psi | \hat{n} | \psi \rangle) = \\ &= |g|^2 (1 + \bar{n}) \end{aligned}$$

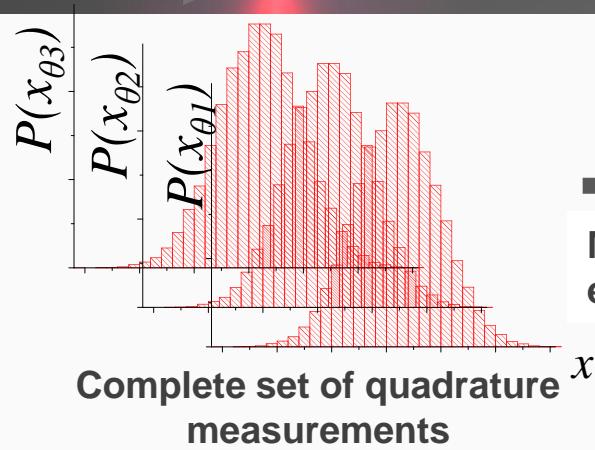
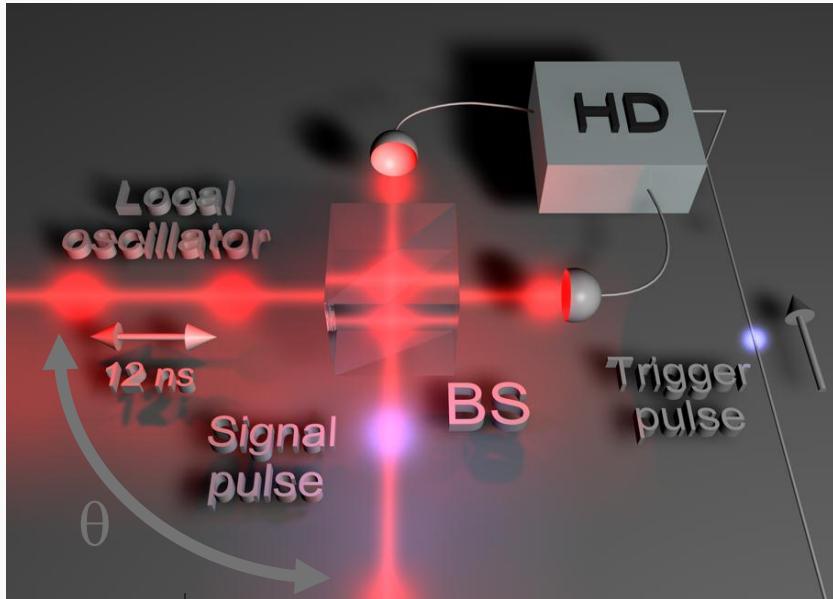


... whenever an idler photon is detected, the signal is prepared in a *single-photon-excited* version of the initial state

# Experimental single-photon addition



# Ultrafast homodyne detection & quantum tomography



$$\{P(x_\theta)\}_{\theta \in [0 \div \pi]} \xrightarrow{\text{Maximum likelihood estimation}} \rho_{nm} = \langle n | \hat{\rho} | m \rangle$$



Wigner function

Gated time-domain acquisition  
Ultra-high bandwidth ( $\sim 100$  MHz)  
Low electronic noise ( $S/N > 10$  dB)  
High subtraction efficiency  
( $\sim 60$  dB @ 82 MHz)

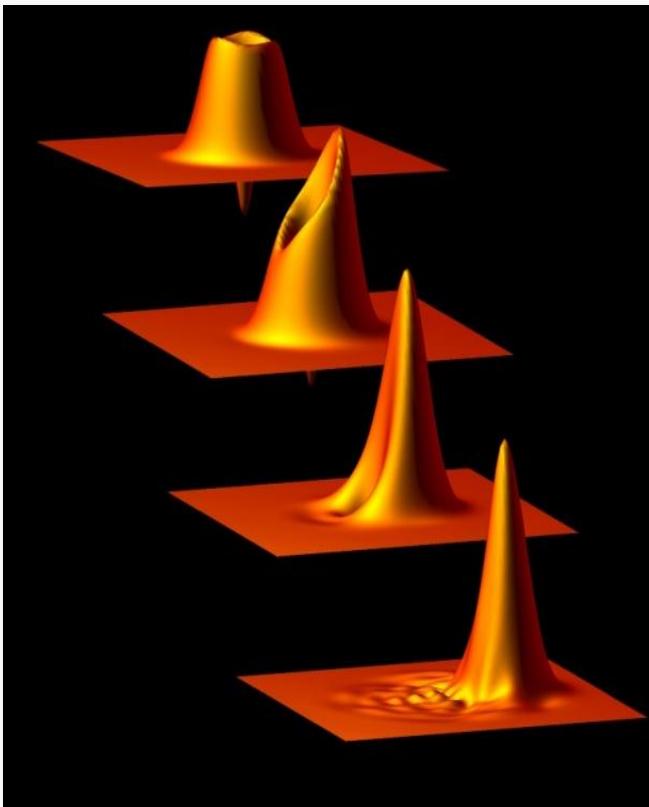
A. Zavatta, M. Bellini, P.L. Ramazza, F. Marin, F.T. Arecchi, J. Opt. Soc. Am. B 19, 1189 (2001)

Density matrix elements

$$\rho_{nm} = \langle n | \hat{\rho} | m \rangle$$

# Adding a single photon to a state of light

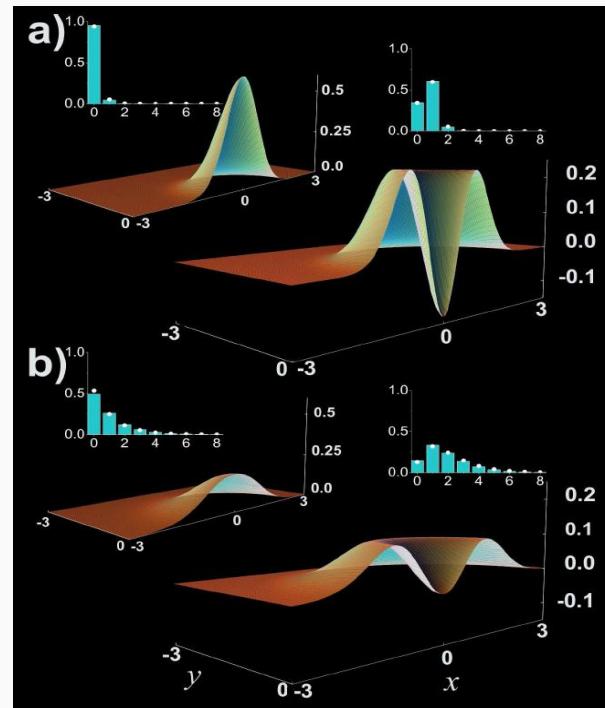
SPACS



**Particle-to-wave transition  
Spontaneous-to-stimulated emission**

A. Zavatta, S. Viciani, M. Bellini, *Science*, 306, 660 (2004), *PRA* 72, 023820 (2005).

SPATS



## Test of criteria for nonclassicality

A. Zavatta, V. Parigi, M. Bellini, *PRA* 75, 052106 (2007)

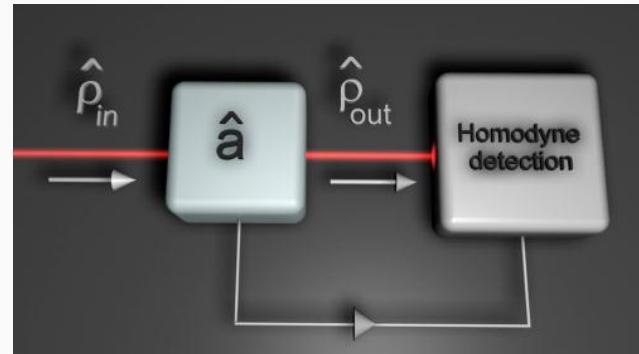
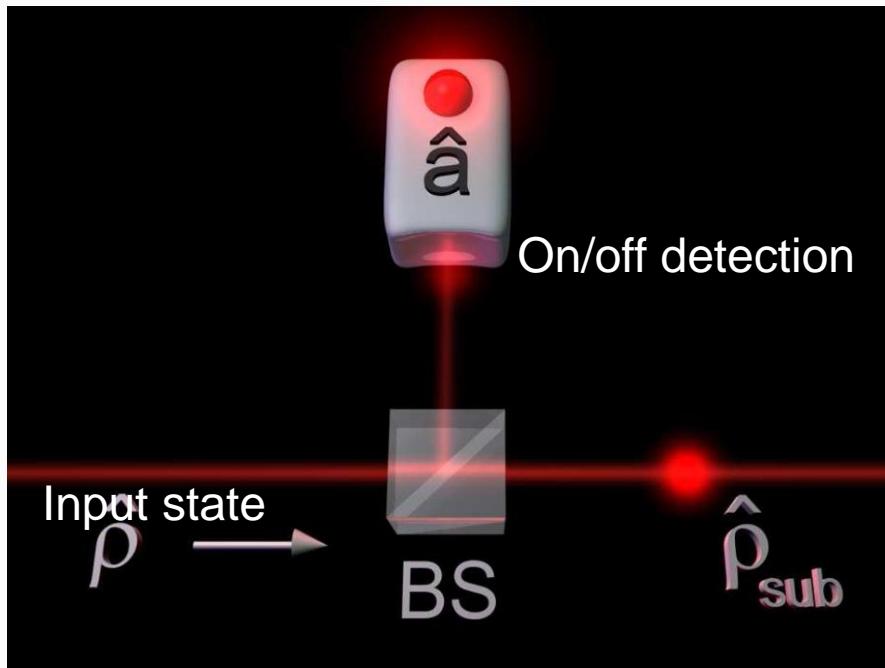
## Reconstruction of nonclassical P-function

T. Kiesel, W. Vogel, V. Parigi, A. Zavatta, M. Bellini, *PRA* 78, 021804(R) (2008)

## Nonclassical quasiprobabilities

T. Kiesel, W. Vogel, M. Bellini, A. Zavatta, *PRA* 83, 032116 (2011)

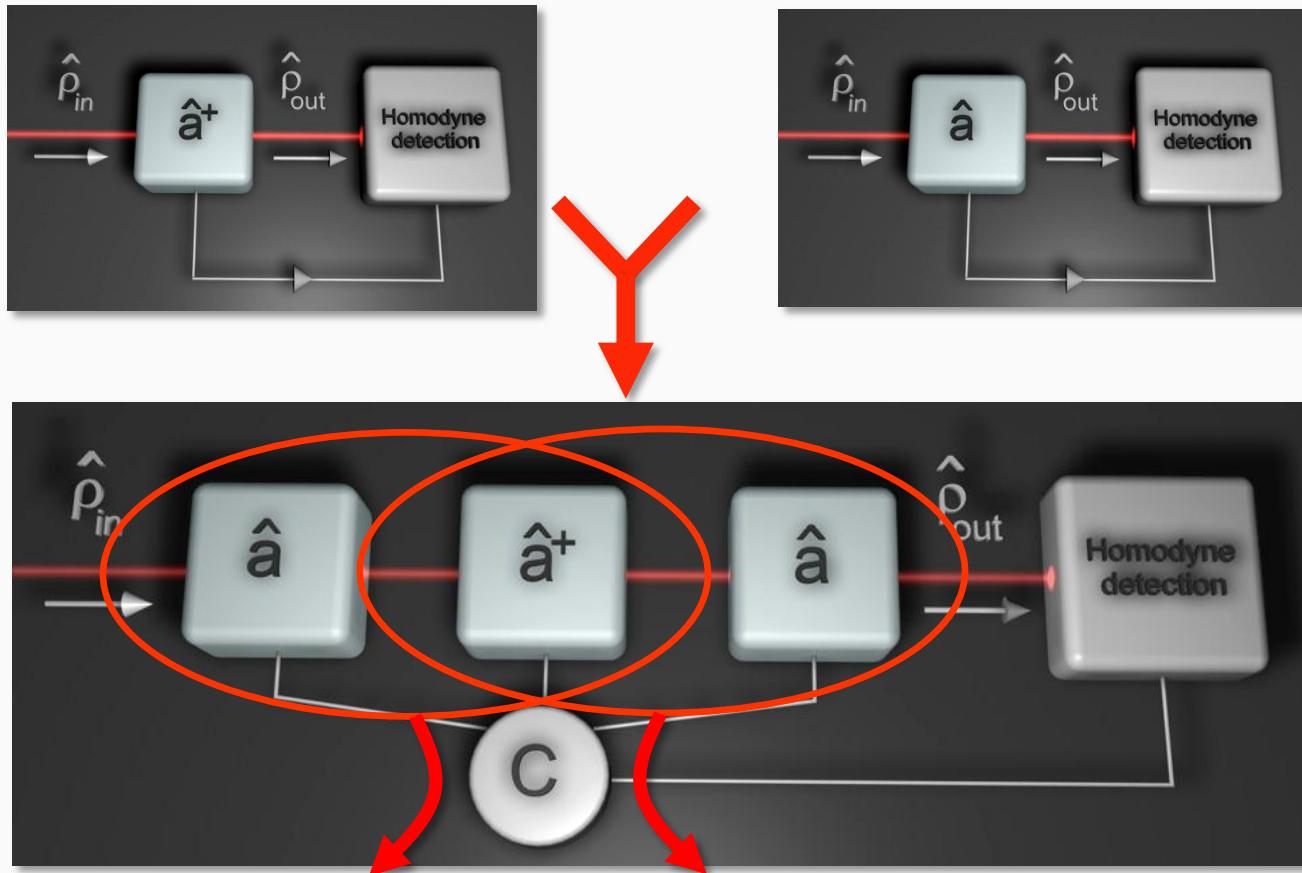
# How to “subtract” a single photon


 $\hat{a}$ 

Faithful implementation of the annihilation operator for:

- Low BS reflectivity
- Low photon numbers

# Combining quantum operators

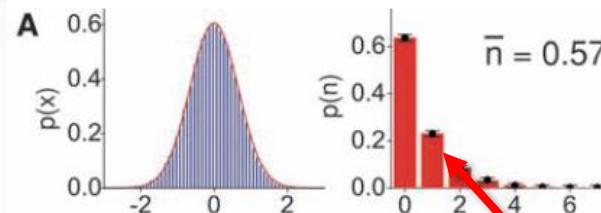


$$\hat{a}^\dagger \hat{a}$$

$$\hat{a} \hat{a}^\dagger$$

# Homodyne data and reconstructions

1



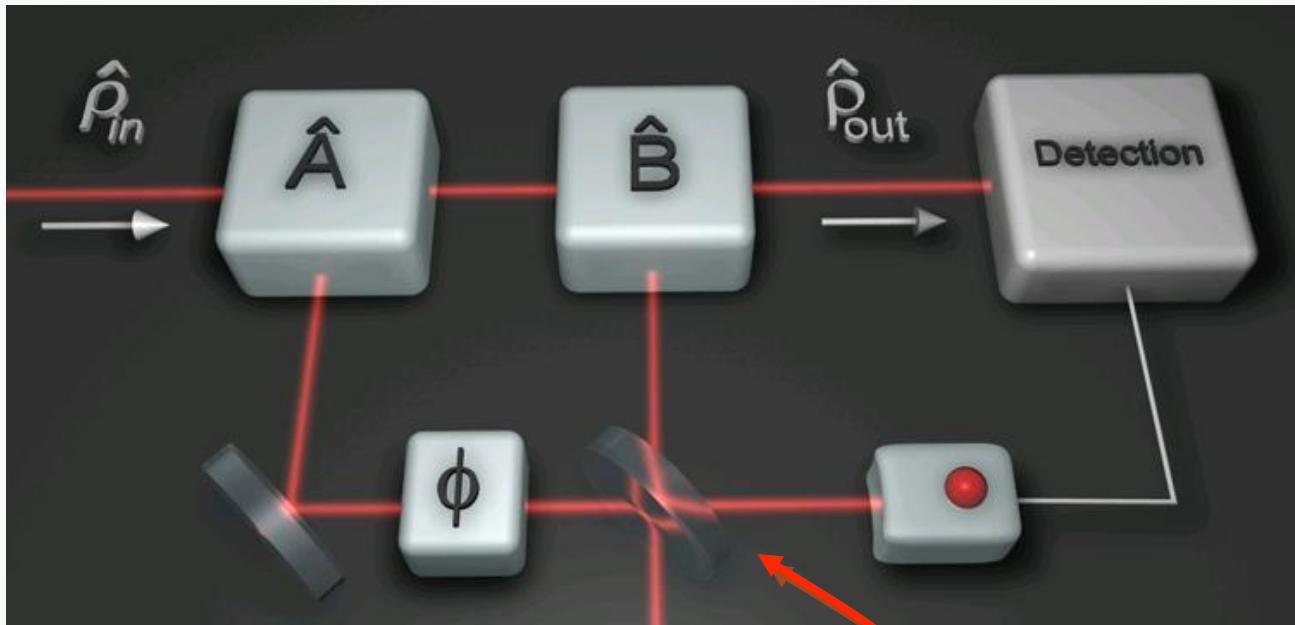
Limited preparation efficiency ( $\xi=92\%$ ) in the photon-addition stage adds small contaminations

$$[\hat{a}, \hat{a}^\dagger] \neq 0$$

V. Parigi, A. Zavatta, M.S. Kim, MB  
*Science* **317**, 1890 (2007)

Clearly different  
final states

# Superpositions of quantum operators



$$|\alpha| \hat{A} + e^{i\phi} |\beta| \hat{B}$$

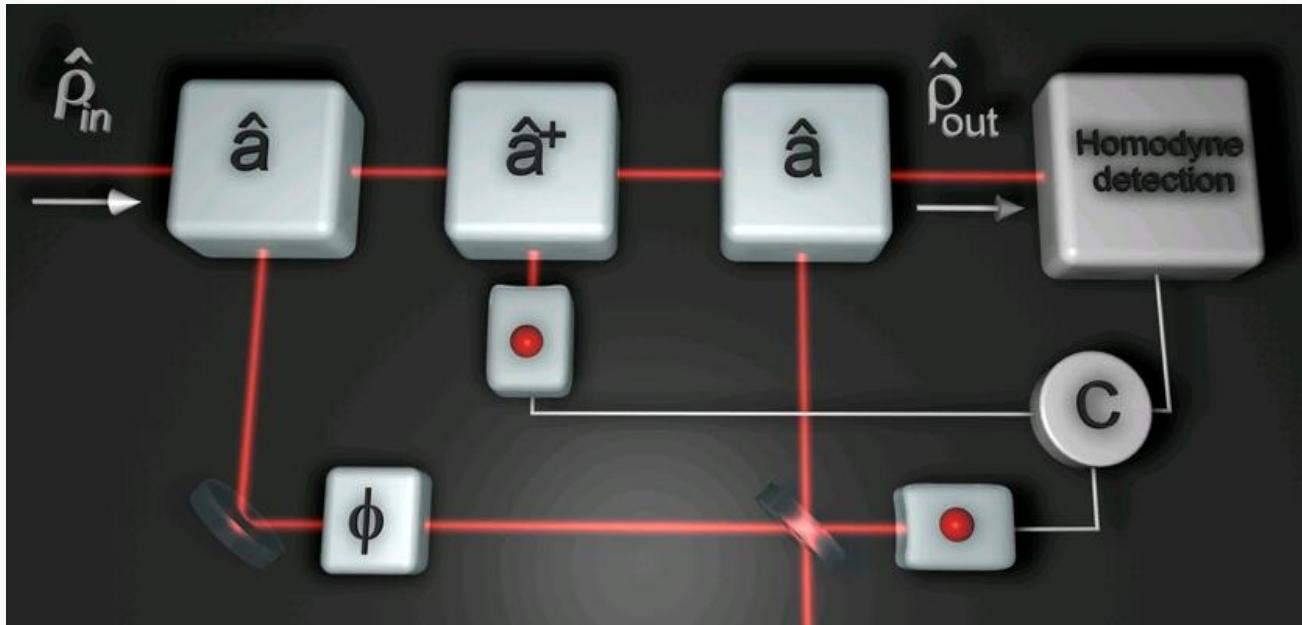
Erases the information about  
the origin of a “click”

Arbitrary superpositions of operators  
can be implemented

Apply to any state

Arbitrary state  
superposition

# Complete test of commutation relations



$$\hat{a}\hat{a}^\dagger - e^{i\phi} \hat{a}^\dagger \hat{a}$$

$\downarrow$   
 $\phi = 0$

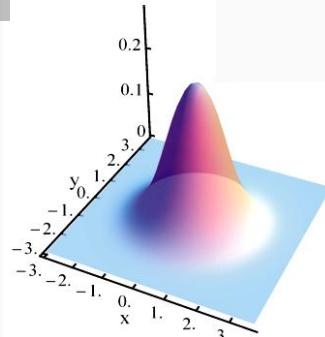
$$[\hat{a}, \hat{a}^\dagger] = \hat{a}\hat{a}^\dagger - \hat{a}^\dagger \hat{a} = 1$$

This complex superposition of operations should do nothing to the state !!

# Testing commutation rules

$$\hat{a}\hat{a}^\dagger - e^{i\phi} \hat{a}^\dagger \hat{a}$$

Initial thermal state



Experimentally reconstructed Wigner functions

$$F = |Tr \sqrt{\sqrt{\hat{\rho}_{in}} \hat{\rho}_{out} \sqrt{\hat{\rho}_{in}}}|^2 > 0.99$$

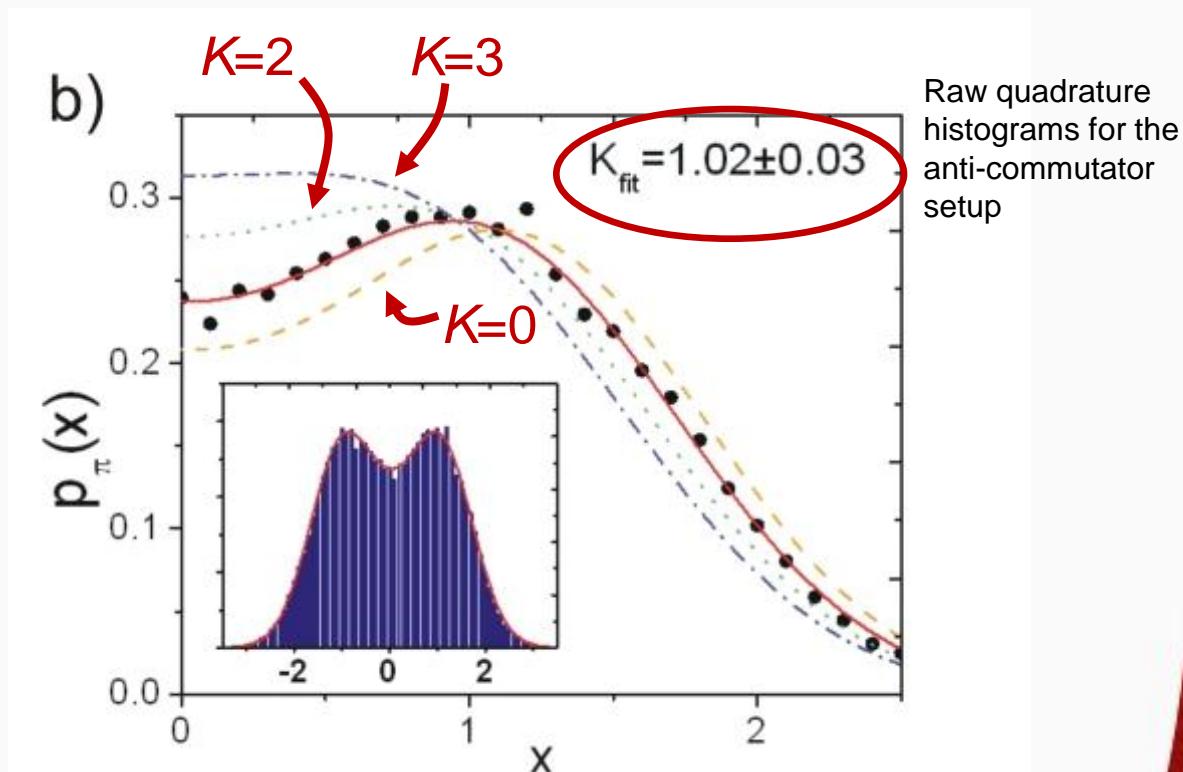
# Quantitative test of commutation rules

$$[\hat{a}, \hat{a}^\dagger] = K\mathbf{1}$$

then the anti-commutator would correspond to

$$2\hat{a}^\dagger\hat{a} + K\mathbf{1}$$

The final state strongly depends on the exact value of  $K$  and can be experimentally tested



The superposition scheme works well and it can be used for QIP

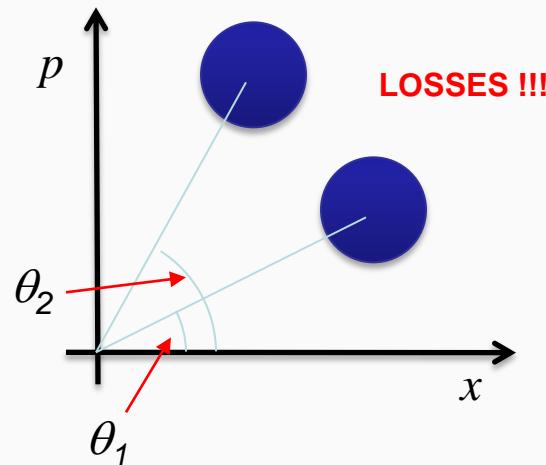
# Phase-insensitive noiseless amplification

$$|\alpha\rangle \rightarrow |\alpha e^{i\theta}\rangle$$

Encoding information in the phase of a coherent state

$$V(\theta_{\text{est}}) = \frac{1}{4|\alpha|^2}$$

Standard quantum limit on phase measurements



Phase-insensitive, noiseless, linear amplification of coherent states

$$|\alpha\rangle \rightarrow |g\alpha\rangle$$

Unfortunately, this is not allowed by the linearity and unitary evolution of Quantum Mechanics!

- ✖ Clone quantum states
- ✖ Violation of Heisenberg uncertainty principle
- ✖ Send superluminal information

# Non-deterministic noiseless amplification

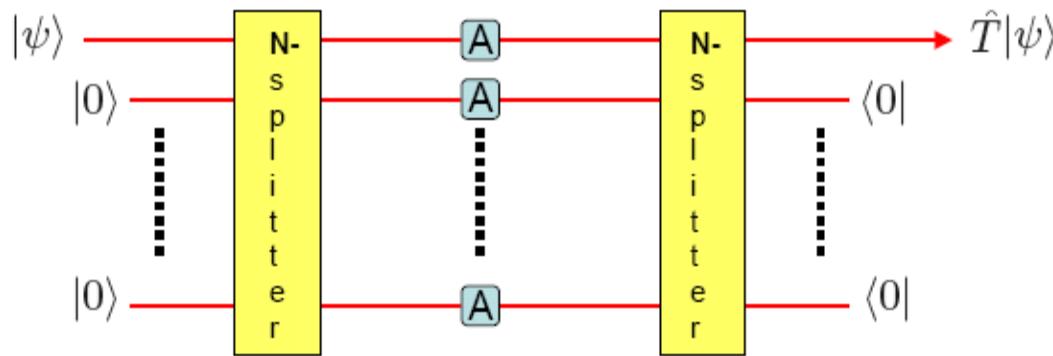
$$|\alpha\rangle \rightarrow |g\alpha\rangle$$

Only a non-deterministic implementation is possible

$$|\alpha\rangle\langle\alpha| \rightarrow \rho(\alpha) = P|g\alpha\rangle\langle g\alpha| + (1 - P)|0\rangle\langle 0|$$

## Nondeterministic Noiseless Linear Amplification of Quantum Systems

T.C.Ralph<sup>1</sup> and A.P.Lund<sup>1,2</sup>,





# Heralded noiseless amplifiers

LETTERS

PUBLISHED ONLINE: 28 MARCH 2010 | DOI: 10.1038/NPHOTON.2010.35

nature  
photronics

## Heralded noiseless linear amplification and distillation of entanglement

G. Y. Xiang<sup>1</sup>, T. C. Ralph<sup>2</sup>, A. P. Lund<sup>1,2</sup>, N. Walk<sup>2</sup> and G. J. Pryde<sup>1\*</sup>

PRL 104, 123603 (2010)

PHYSICAL REVIEW LETTERS

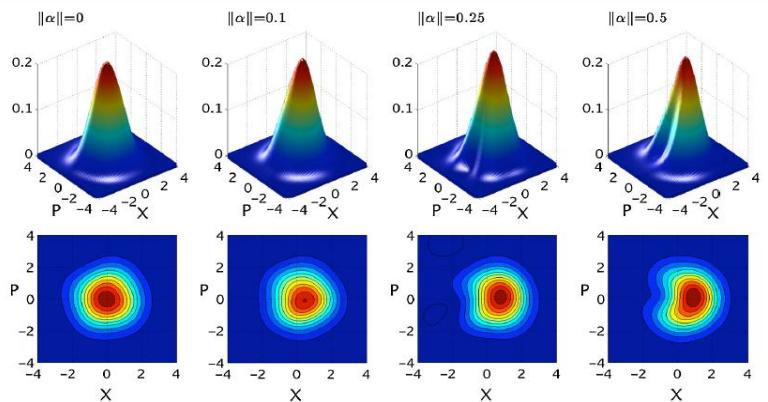
week ending  
26 MARCH 2010

### Implementation of a Nondeterministic Optical Noiseless Amplifier

Franck Ferreyrol, Marco Barbieri, Rémi Blandino, Simon Fossier, Rosa Tualle-Brouri, and Philippe Grangier

Groupe d'Optique Quantique, Laboratoire Charles Fabry, Institut d'Optique, CNRS, Université Paris-Sud, Campus Polytechnique,  
RD 128, 91127 Palaiseau cedex, France

(Received 10 December 2009; published 24 March 2010)



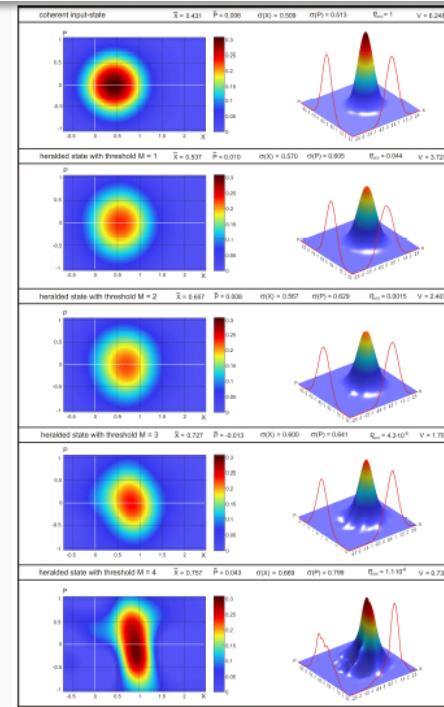
nature  
physics

LETTERS

PUBLISHED ONLINE: 15 AUGUST 2010 | DOI: 10.1038/NPHYS1743

## Noise-powered probabilistic concentration of phase information

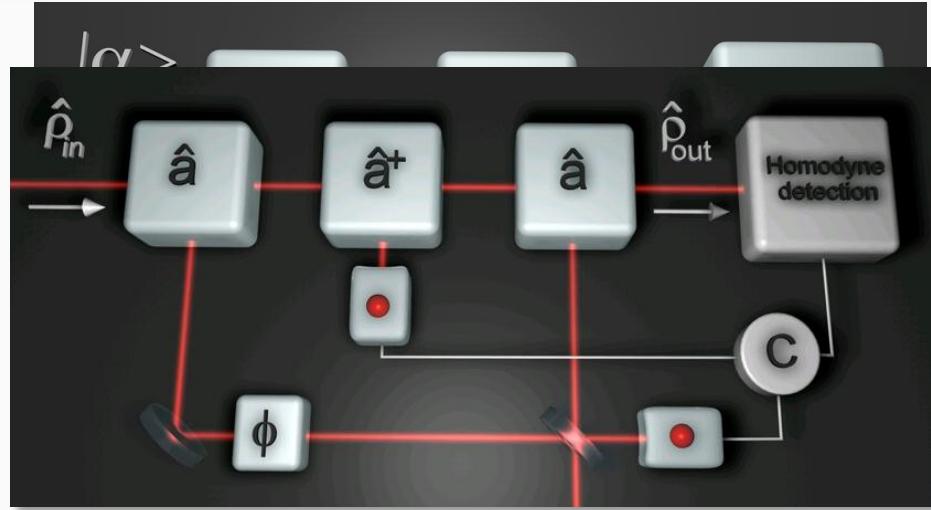
Mario A. Usuga<sup>1,2†</sup>, Christian R. Müller<sup>1,3†</sup>, Christoffer Wittmann<sup>1,3</sup>, Petr Marek<sup>4</sup>, Radim Filip<sup>4</sup>,  
Christoph Marquardt<sup>1,3</sup>, Gerd Leuchs<sup>1,3</sup> and Ulrik L. Andersen<sup>2\*</sup>



# Noiseless amplification by addition & subtraction

$$\hat{G} = (g - 2)\hat{a}^\dagger \hat{a} + \hat{a} \hat{a}^\dagger$$

$$\hat{G}_{g=2} = \hat{a} \hat{a}^\dagger$$



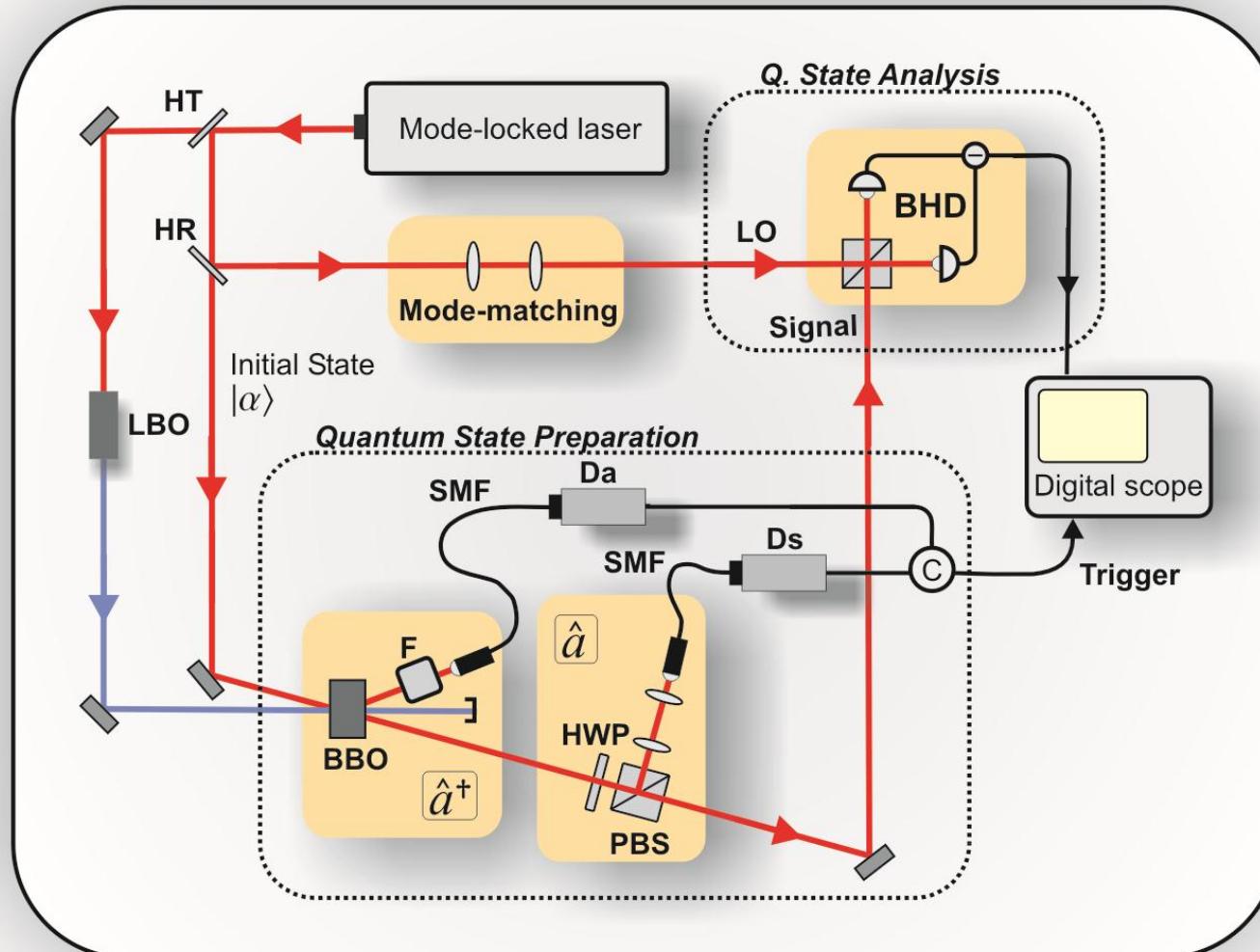
$$|\alpha| \ll 1$$

$$|\alpha\rangle \approx |0\rangle + \alpha |1\rangle$$

$$\hat{a} \hat{a}^\dagger |\alpha\rangle \approx \hat{a} \hat{a}^\dagger (|0\rangle + \alpha |1\rangle) = \hat{a} (|1\rangle + \sqrt{2}\alpha |2\rangle) = |0\rangle + 2\alpha |1\rangle \approx |2\alpha\rangle$$

The final state is not truncated to the  $|1\rangle$  term

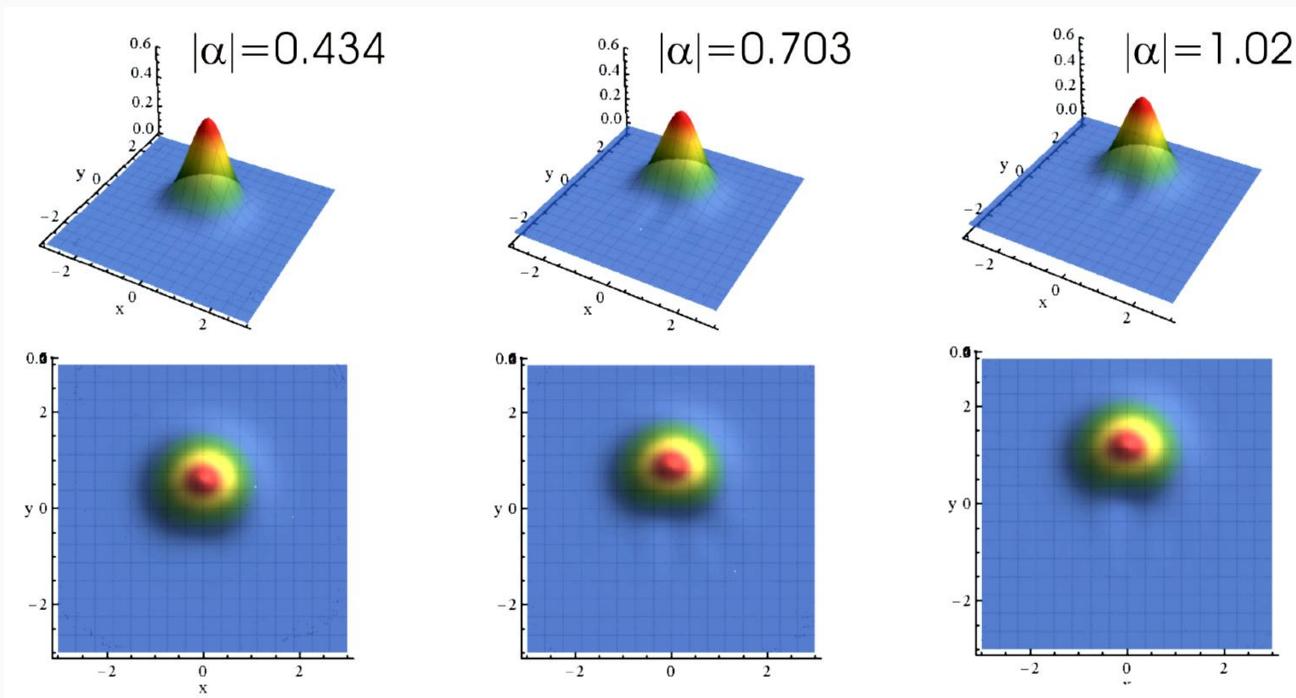
# Experimental setup



# High fidelity quantum amplification

$$\hat{G}_{g=2} = \hat{a}\hat{a}^\dagger$$

Reconstructed Wigner functions for the amplified coherent states:



# High fidelity quantum amplification

Effective amplitude gain

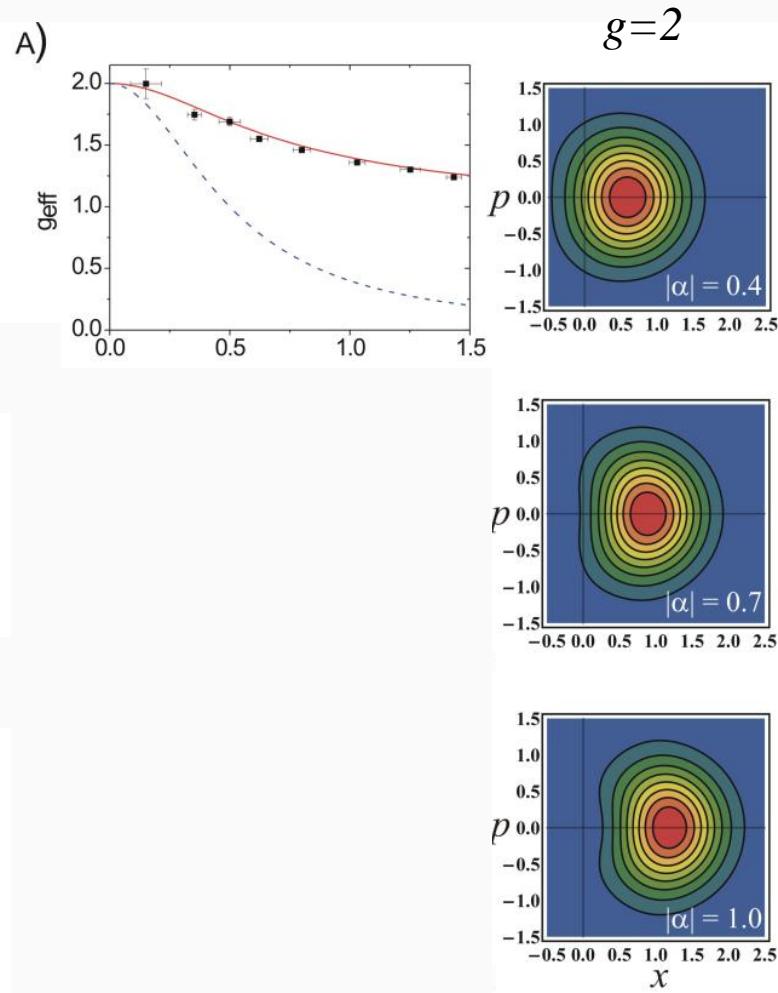
$$g_{eff} = \frac{\langle x_{amp} \rangle}{\langle x_{in} \rangle}$$

Fidelity

Distortions compared to the ideal coherent state of double amplitude

Noise

How much noise is added in the process?



# Variable-gain amplifier

$$\hat{G}_{g=2} = \hat{a}\hat{a}^\dagger$$

Is just a particular case of a general,  
variable-gain, noiseless amplifier

$$\hat{G} = (g - 2)\hat{a}^\dagger\hat{a} + \hat{a}\hat{a}^\dagger$$



Amplitude gain

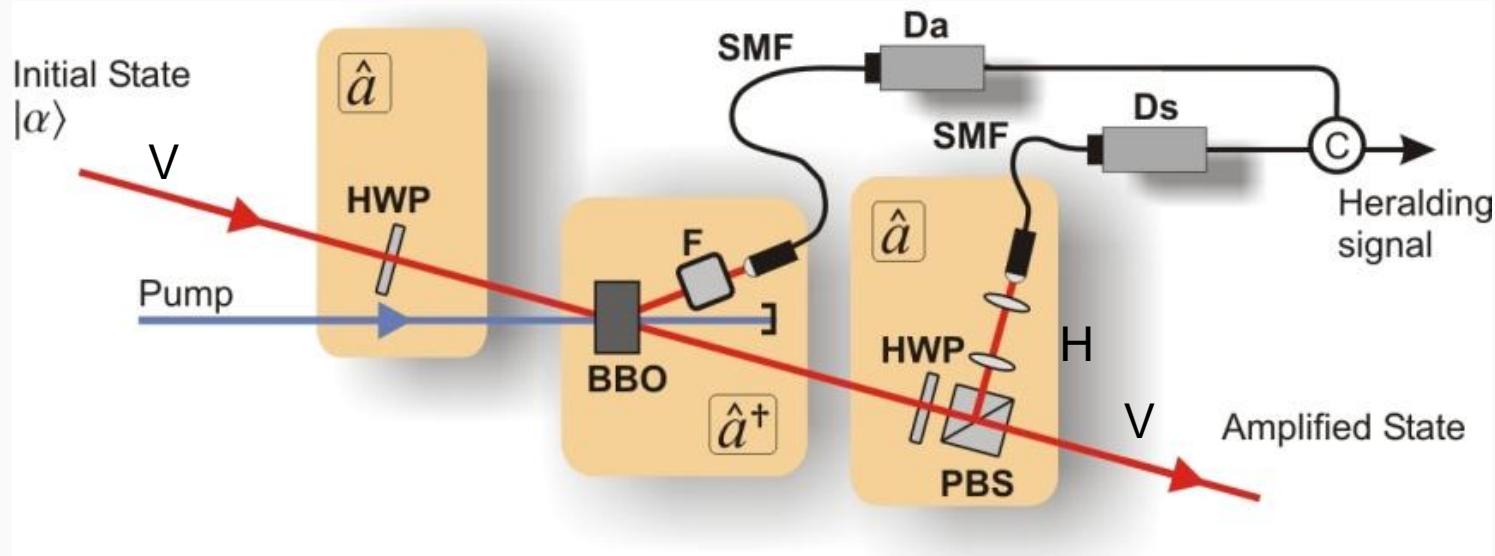
J. Fiurasek, *PRA* **80**, 053822 (2009)

Need a way to produce coherent superpositions of quantum operators

# Variable-gain noiseless amplifier

Superposition of two sequences of operators:

$$\hat{G} = (g - 2)\hat{a}^\dagger \hat{a} + \hat{a} \hat{a}^\dagger$$



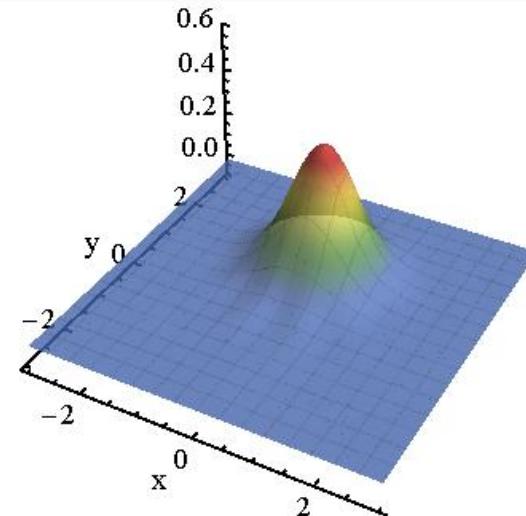
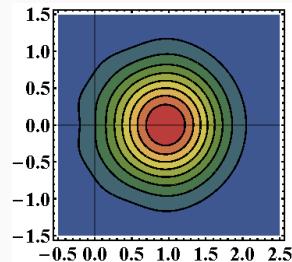
The two HWPs are rotated by very small angles.

# Variable-gain noiseless amplifier

Input coherent state  $|\alpha| \approx 1$

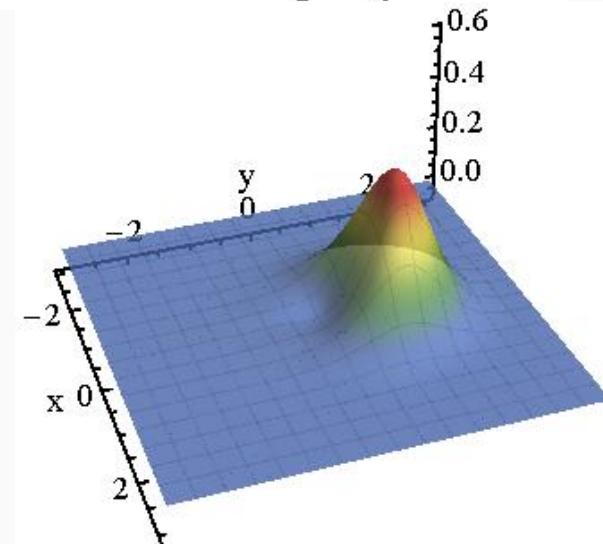
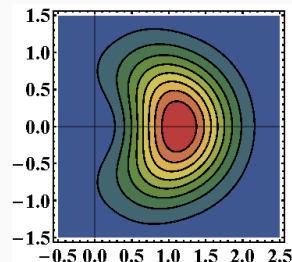
$$g = 1 \quad \hat{a}\hat{a}^\dagger - \hat{a}^\dagger \hat{a}$$

Commutator



$$g = 3 \quad \hat{a}\hat{a}^\dagger + \hat{a}^\dagger \hat{a}$$

Anti-Commutator



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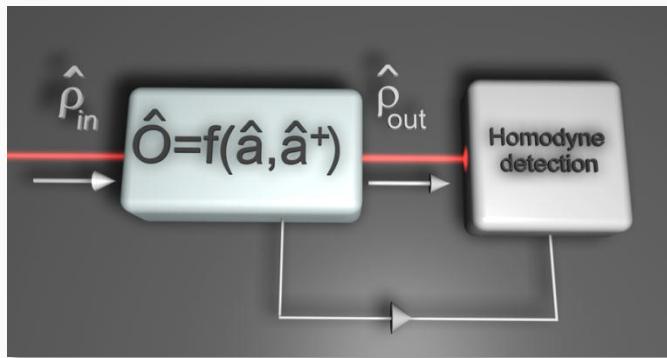
- Single-photon addition and subtraction
- Sequences and superpositions of quantum operators
- Direct probing of fundamental quantum rules
- Noiseless amplification

## Part 2

Investigating the mode structure of ultrashort pulsed quantum light states

- Adaptive homodyne detection
- Spectral-temporal shaping of quantum states

# The “shape” of a quantum state

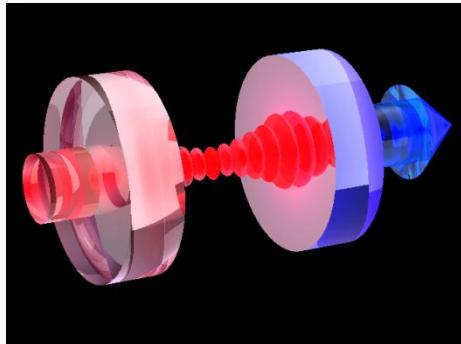


Every operation is performed in a single, well-defined, spatio-temporal mode



The relevant quantum features of the states can only be accessed if the right mode is properly selected and analyzed

## Excitation of a particular spatio-temporal mode

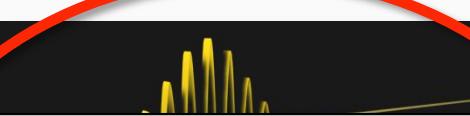


Confined cavity mode

$$|\omega\rangle = \hat{a}^\dagger(\omega) |0\rangle$$



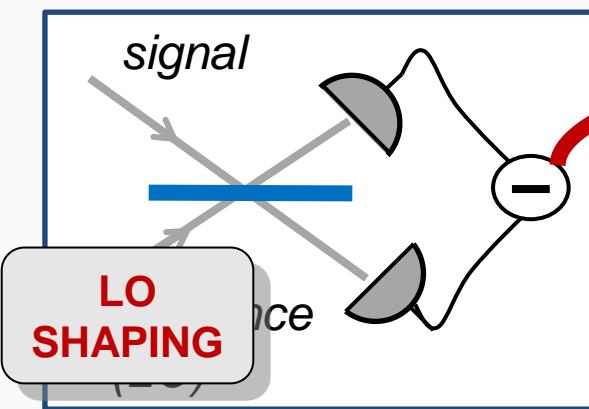
Infinitely-extended  
monochromatric CW mode



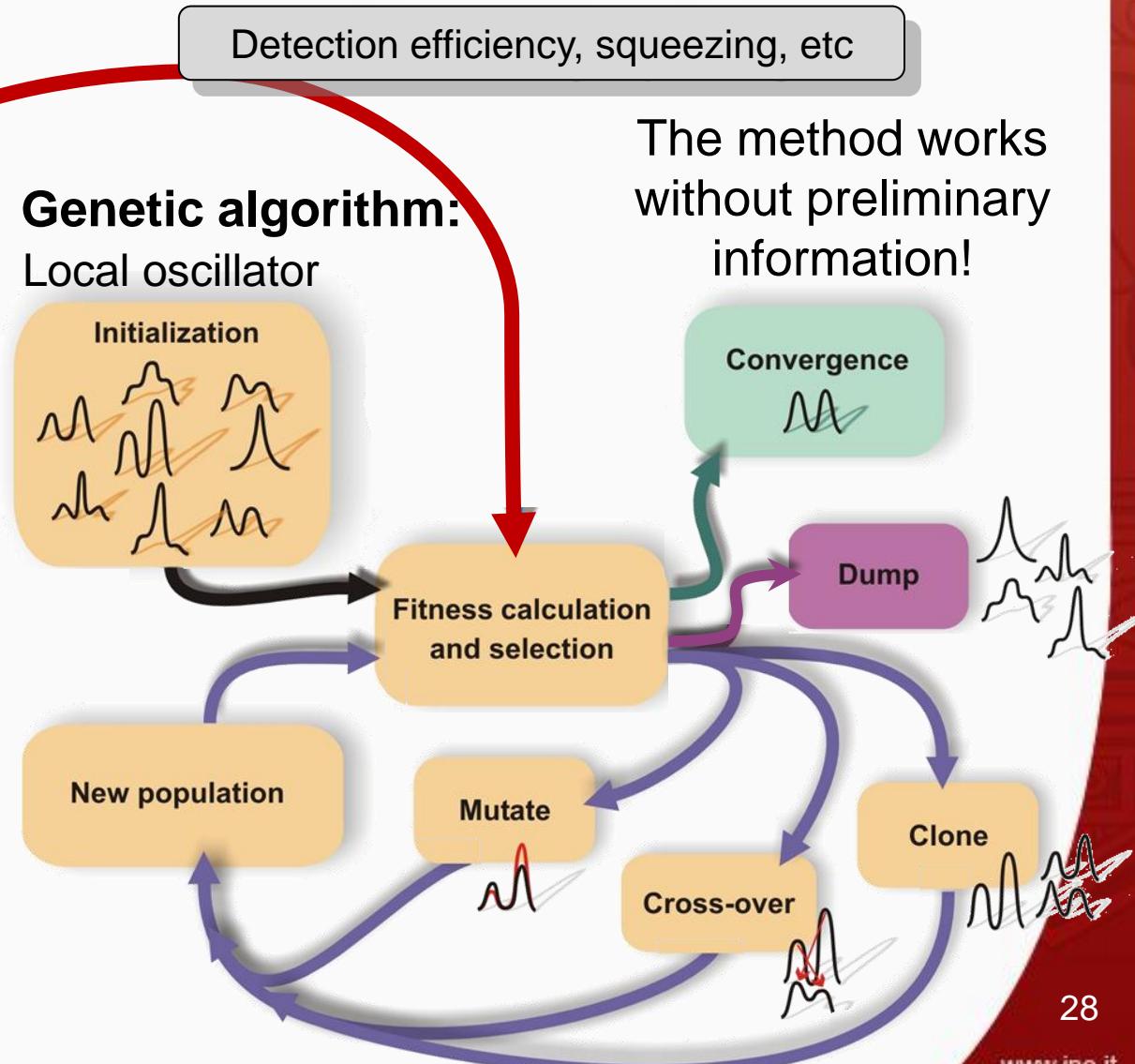
Manipulate and characterize the spectral content of ultrashort (<100 fs) quantum states of light

$$|1\rangle_\Psi = \int d\omega \Psi(\omega) |\omega\rangle$$

# Adaptive measurement scheme

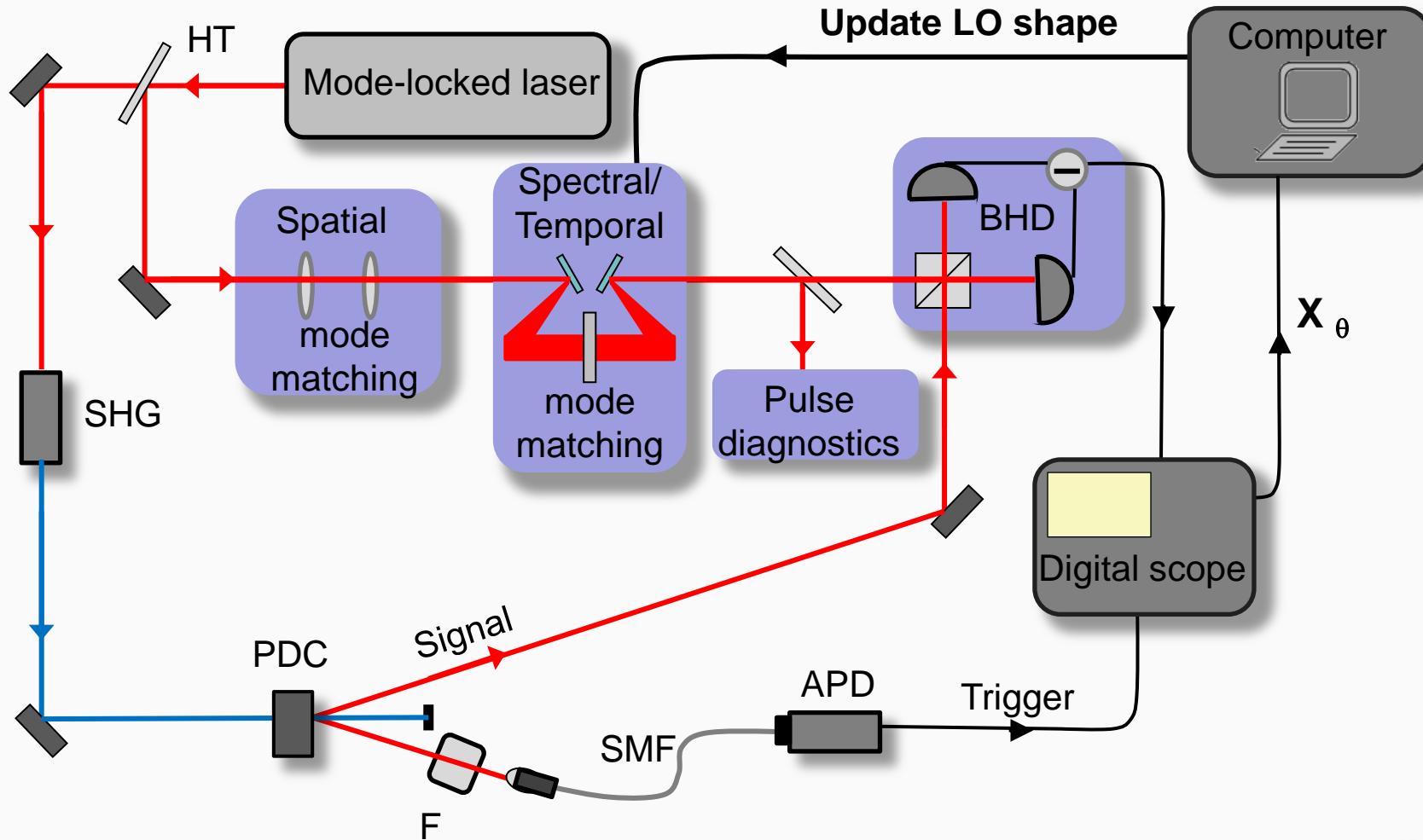


A quantum state can only be efficiently observed if the LO is properly matched in polarization, space, time, spectrum,...

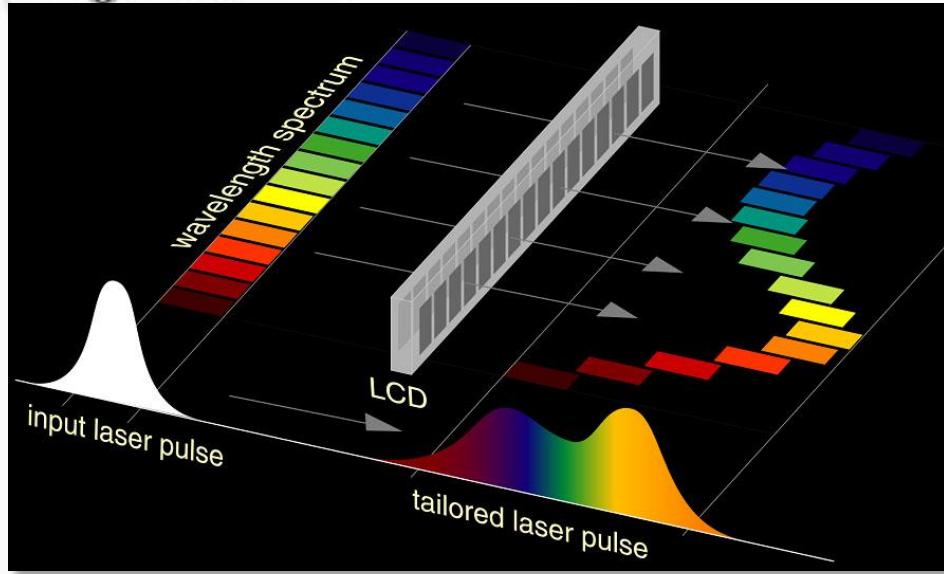


# Measuring the photon wavepacket

82 MHz, @ 800 nm  
 $\Delta\tau \sim 70$  fs,  $\Delta\lambda \sim 10$  nm



# Shaping ultrashort LO pulses



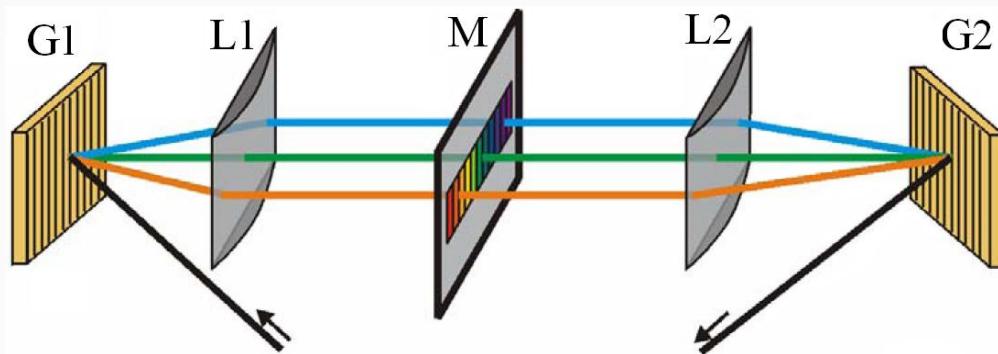
## SLM pulse shaper

One needs to independently modulate each wavelength component in amplitude and phase

Two Spatial Light Modulators

128 pixels each

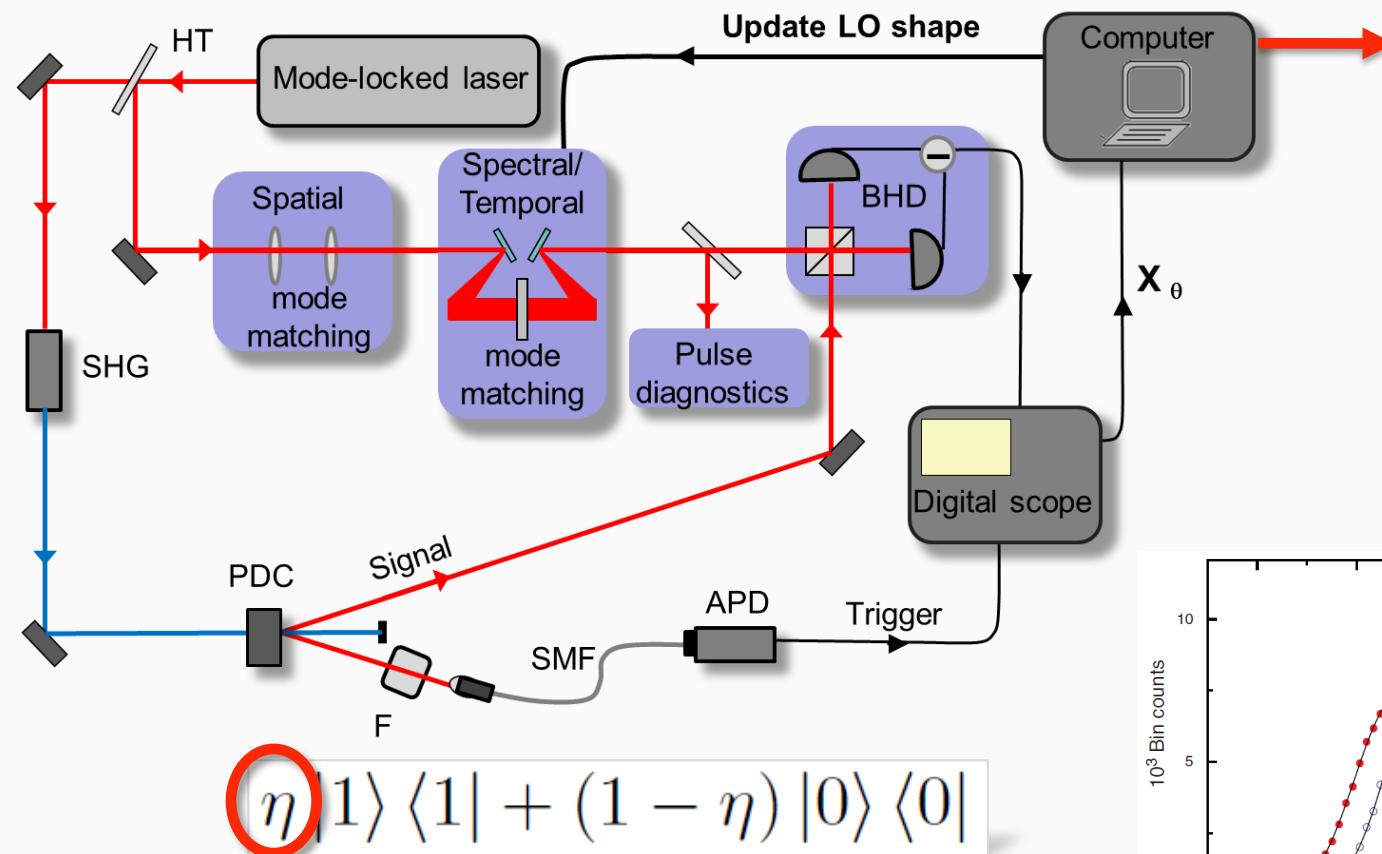
Resolution 0.6 nm





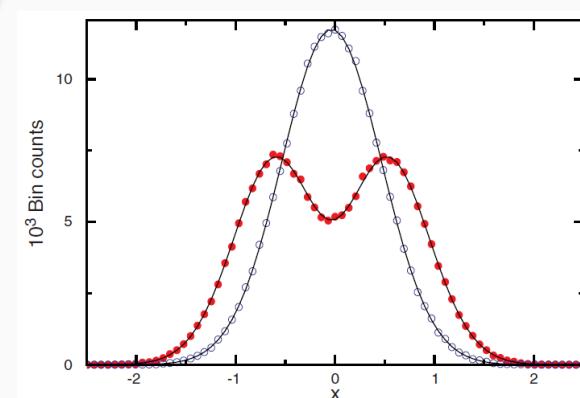
# Searching for the photon shape

82 MHz, @ 800 nm  
 $\Delta\tau \sim 70$  fs,  $\Delta\lambda \sim 10$  nm



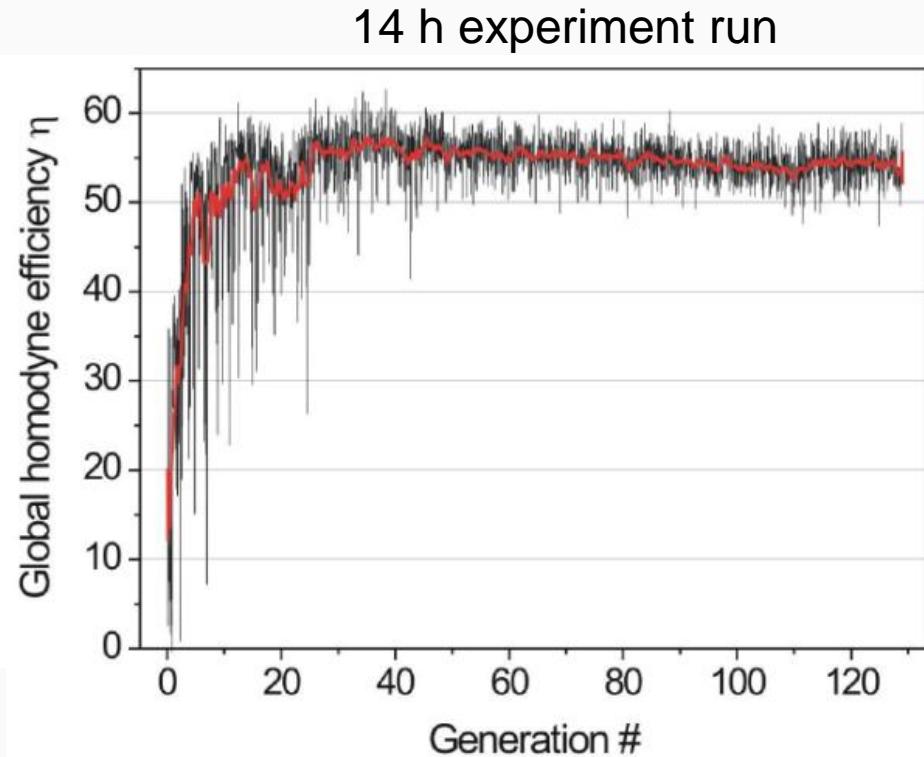
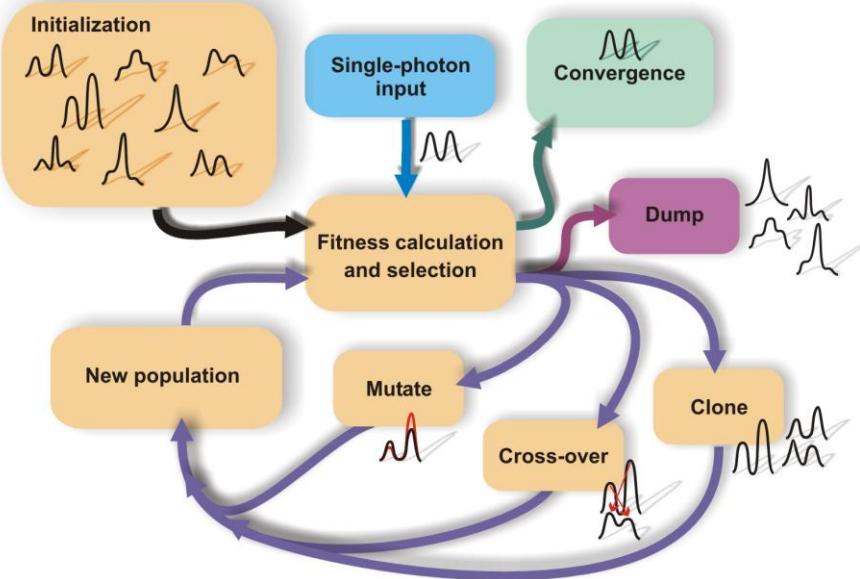
$\eta$  quantifies the amount of pure single photon in the detected mixed state

Calculate the efficiency  $\eta$



Single-photon quadrature distribution

# Evolutionary search of the photon shape

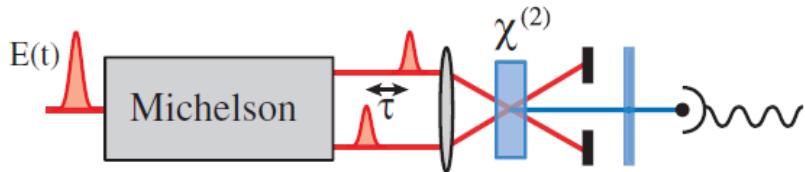


Using the evolutionary algorithm to find the best LO pulse shape

No preliminary information is needed

# Retrieving the LO shape of light (FROG)

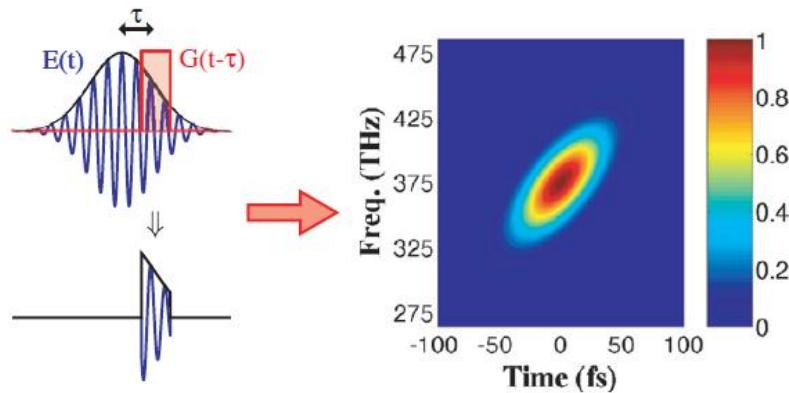
## Autocorrelation



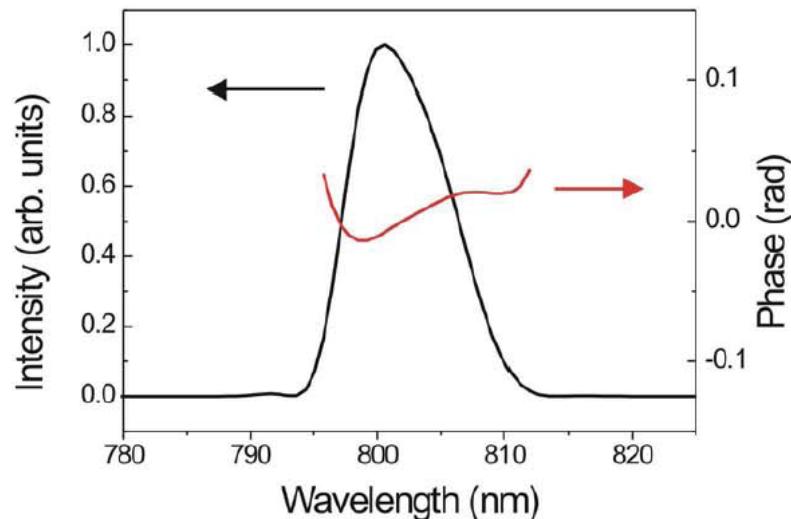
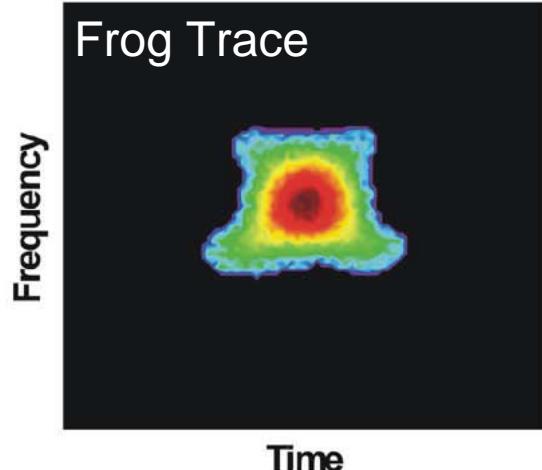
Assuming pulse profile  $\rightarrow$  pulse duration

- A. Monmayrant *et al.*, J. Phys. B **43**, 103001 (2010)  
 I.A. Walmsley and C. Dorrer, Adv. Opt. and Phot. **1**, 308 (2009)

## FROG (Frequency Resolved Optical Gating)

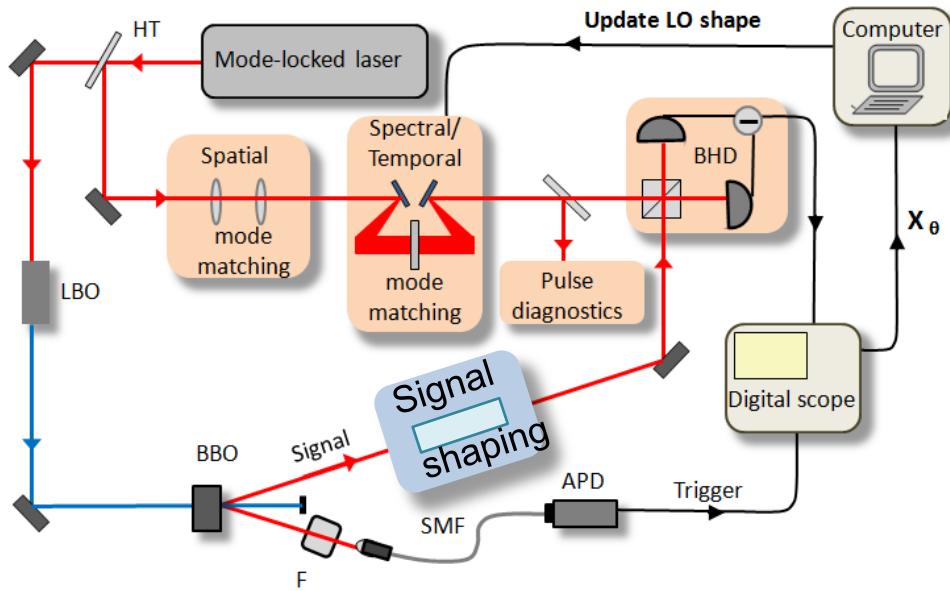


Unmodulated single photon shape





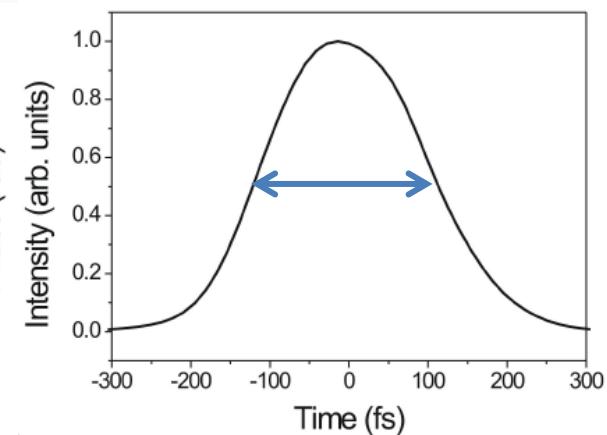
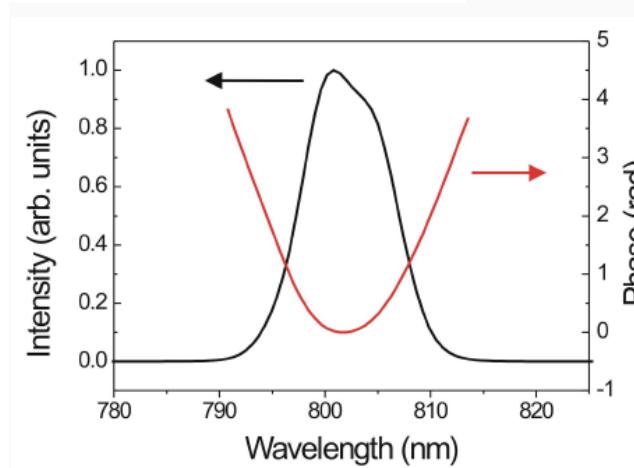
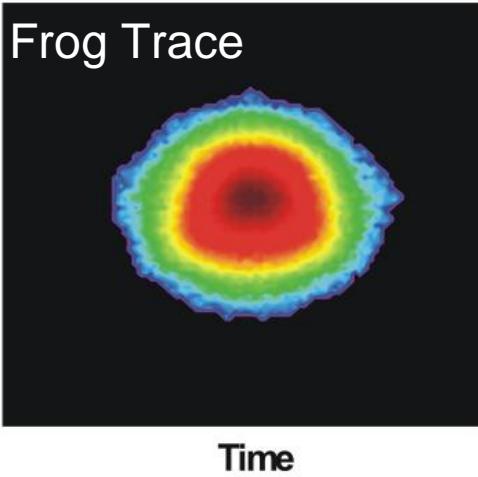
# Introducing linear dispersion



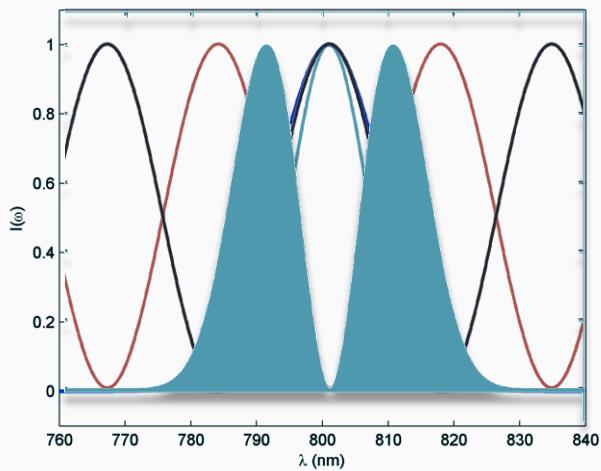
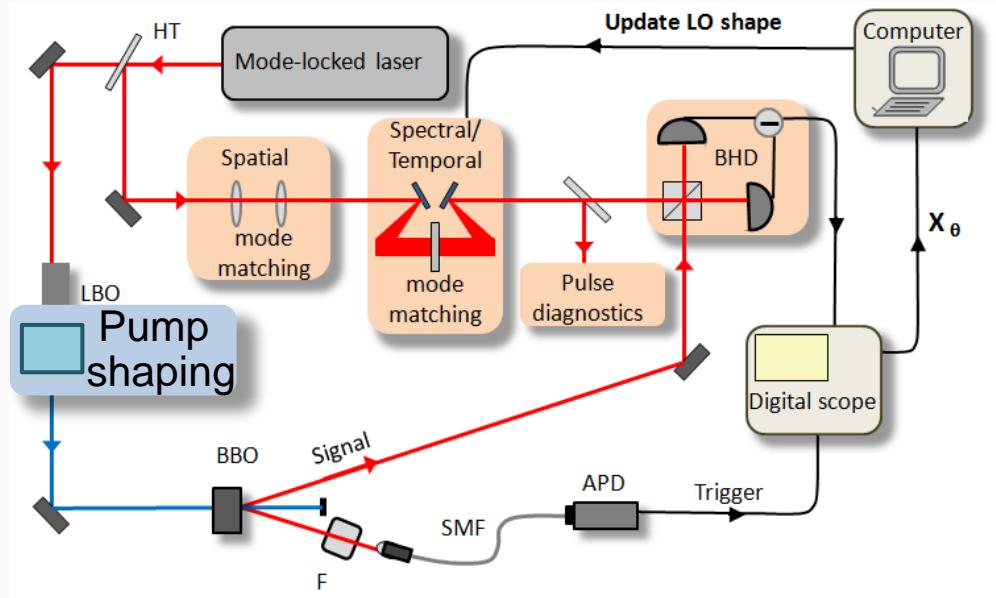
10 cm long BK7 glass

Retrieved mode presents a temporally stretched shape, as expected.

$$\eta = 60\%$$

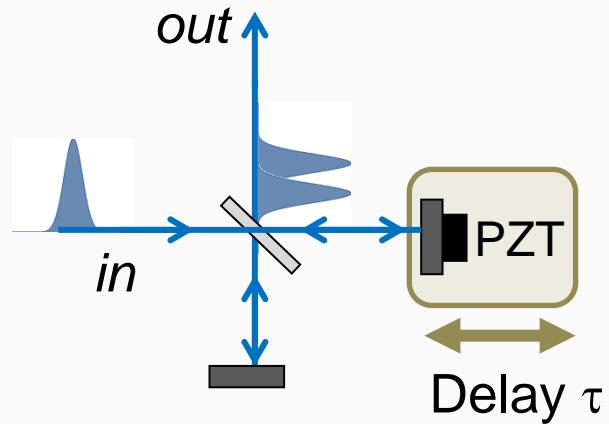


# More complex shapes



Sinusoidal modulation

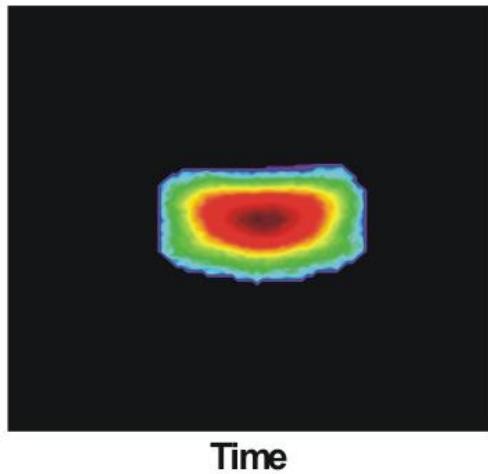
Michelson interferometer



Phase between pulses  
(controlled with PZT)

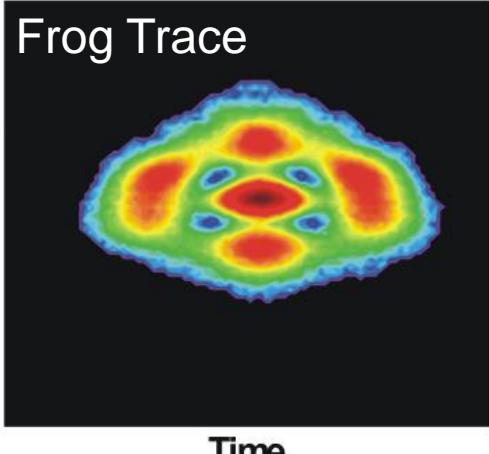
# Results – LO optimization

Frequency



Time

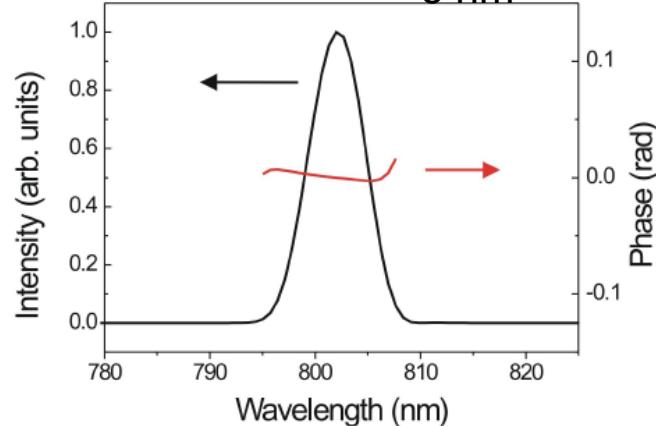
Frequency



Time

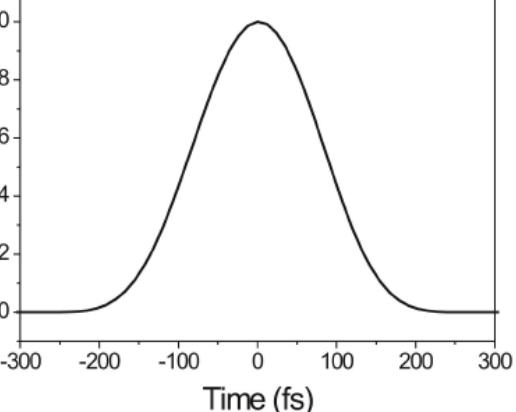
Phase = 0

6 nm

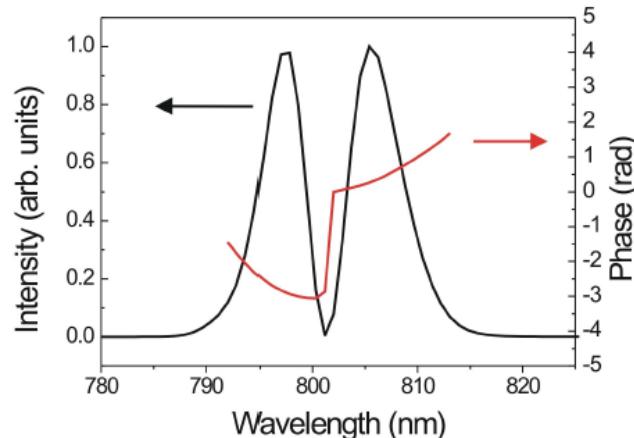


180 fs

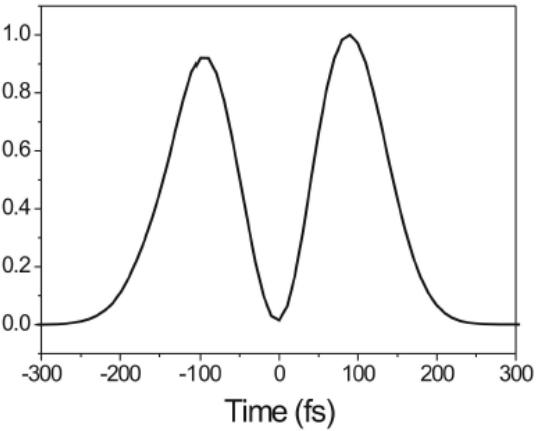
Intensity (arb. units)



Phase =  $\pi$



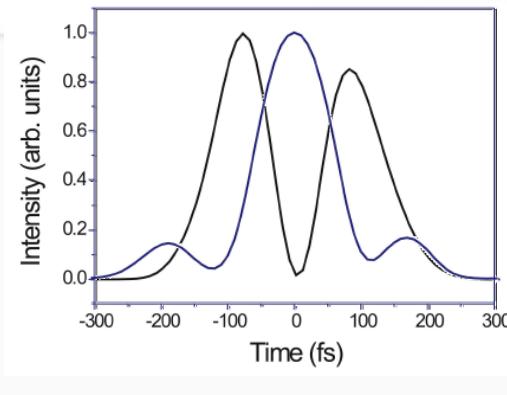
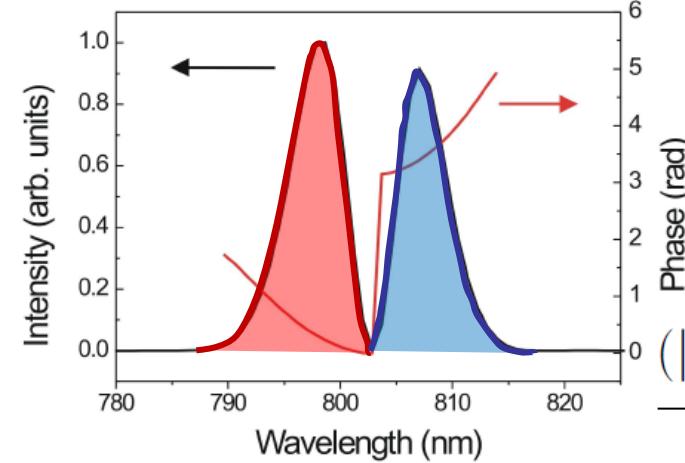
Intensity (arb. units)



# Probing coherence

$\Psi_1$

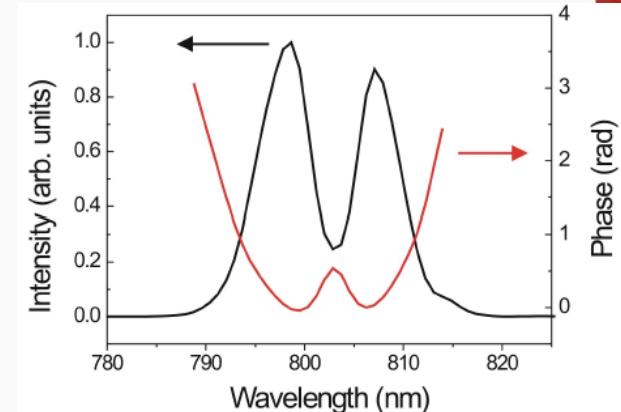
$\Psi_2$



$$(|1\rangle_{\Psi_1}|0\rangle_{\Psi_2} + |0\rangle_{\Psi_1}|1\rangle_{\Psi_2})$$

$$\sqrt{2}$$

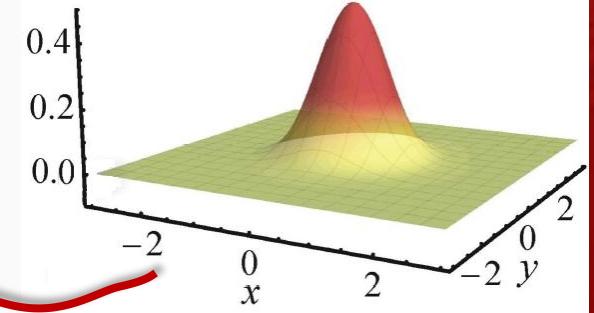
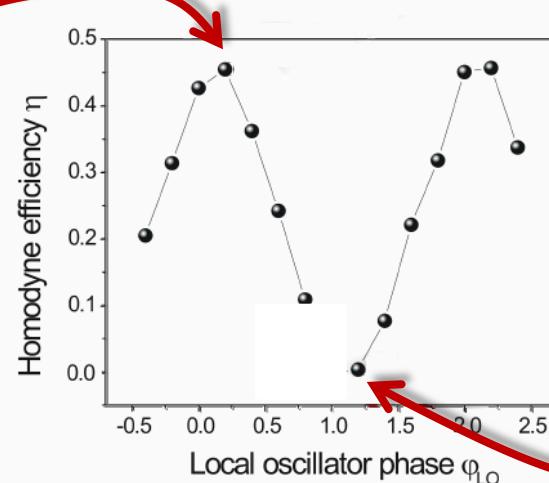
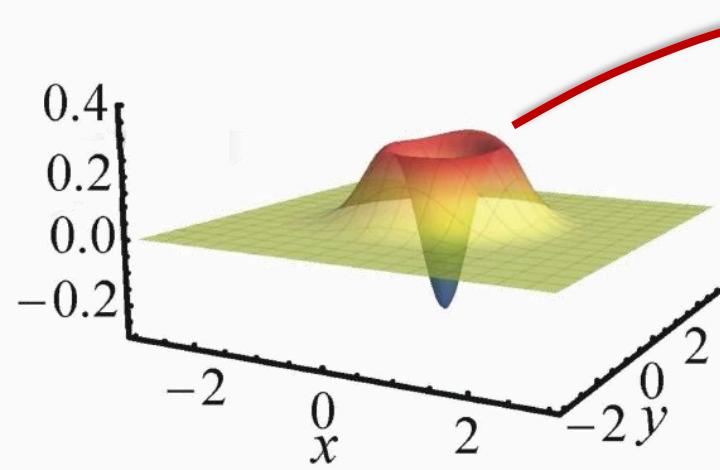
Coherent superposition of two distinct spectral modes



$$(|1\rangle_{\Psi_1}|0\rangle_{\Psi_2} - |0\rangle_{\Psi_1}|1\rangle_{\Psi_2})$$

$$\sqrt{2}$$

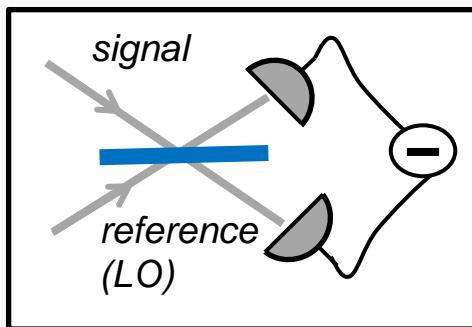
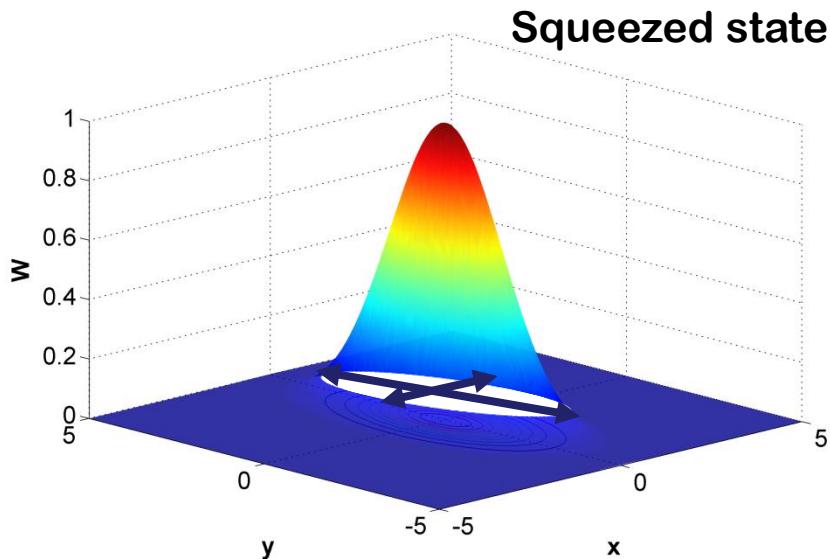
Probe coherence  
using shaped LO mode!



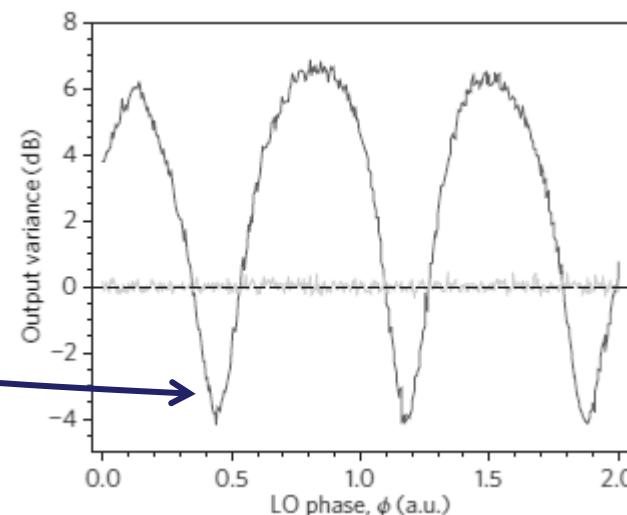
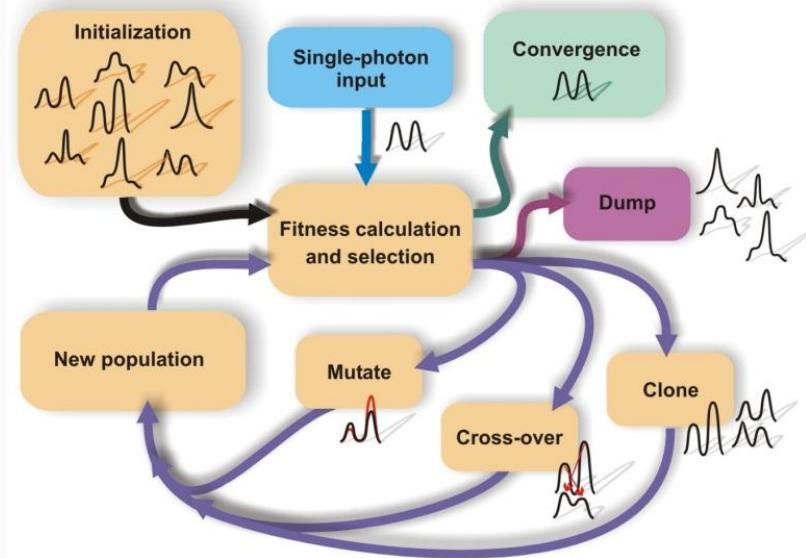
# What about other quantum states of light?

It is only necessary to have a fitness parameter

Can be applied to multiphoton states

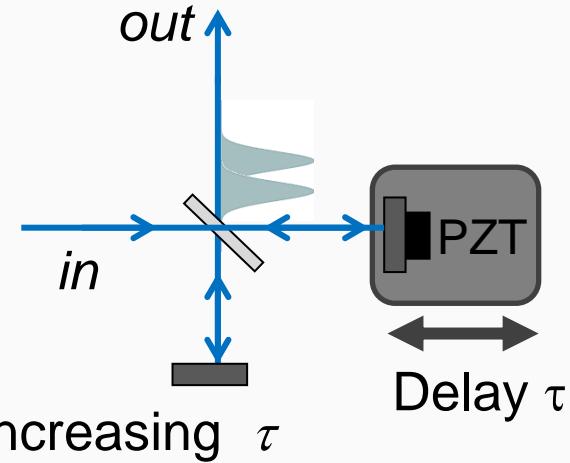


Fitness would be the quantum noise level

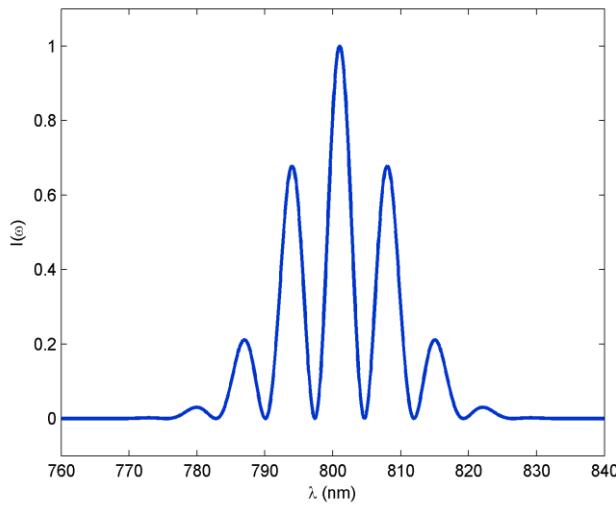


# Perspectives: multimode light

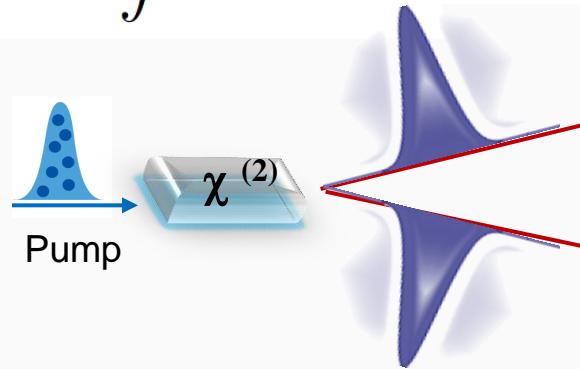
Pump: Michelson interferometer



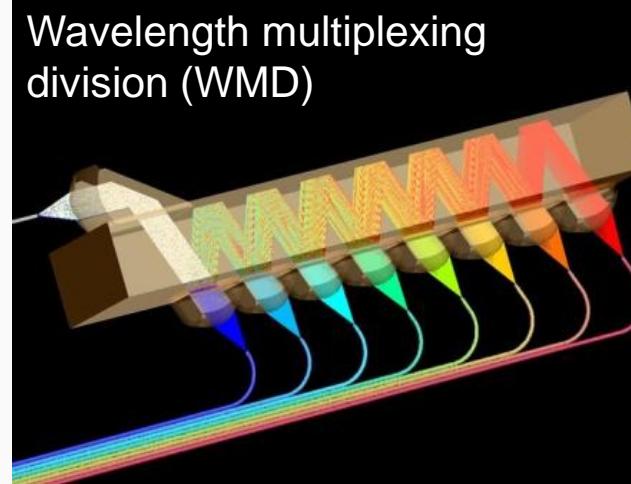
Increasing  $\tau$



$$|1\rangle_{\Psi} = \int d\omega \Psi(\omega) \hat{a}^{\dagger}(\omega) |0\rangle$$

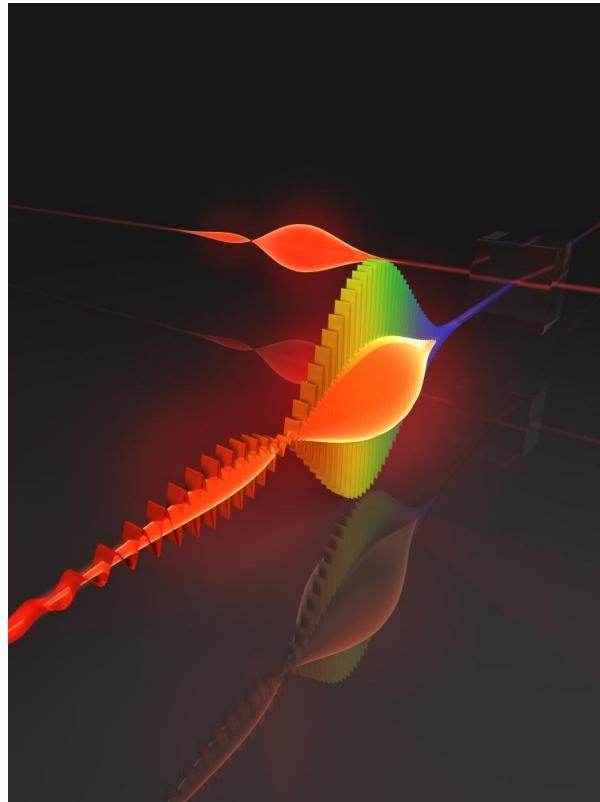


Wavelength multiplexing division (WMD)



Spectro-temporal structure → Platform for encoding quantum information

# Conclusions



## Tools for CV quantum information processing and quantum metrology

Experimental realization of photon creation and annihilation operators

Noiseless linear amplification

Developed an adaptive homodyne measurement scheme to access and probe a rich multimode structure of quantum states

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# Thank you for your attention!

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